# Blunt Injury and Damage: Theory to Interpret Data

# Blunt Injury and Damage: Theory to Interpret Data

**T. P. Hutchinson**

"Blunt Injury and Damage: Theory to Interpret Data" was first published (on the web at BluntInjuryandDamage.com) on 13 November 2018.
This version is dated 21 August 2019.

Blunt Injury and Damage:
Theory to Interpret Data



"Blunt Injury and Damage: Theory to Interpret Data" was first published (on the web at BluntInjuryandDamage.com) on 13 November 2018.
This version is dated 21 August 2019.

## What this book is about

Blunt head injury is perhaps the most common fatal or life-threatening injury.

An important example is a pedestrian's head being struck by a motor vehicle.

Manufactured objects that may strike someone's head, or that are intended to reduce the severity of injury from an impact, are often tested. From that testing, some measurement is obtained that is intended to serve as a proxy for the injury that might occur.

I am dissatisfied with how data from such tests is analysed and reported in research contexts. Thus this book has a particular focus on encouraging improvement of impact test data analysis. It will do that with the aid of some fairly simple theory about the forces acting on colliding objects.

Blunt injury is the chief concern of this book, but there are also chapters on penetrating injury, damage to manufactured items (and protection by packaging), and handling of agricultural produce.

T. P. H.

There is some overlap between the contents of this book and "Road Safety Theory" RoadSafetyTheory.com, which is also by T. P. Hutchinson and also published in 2018. (And there may be similarities with papers by Hutchinson. References to these are given in the usual way.)

Roughly 30 per cent of this book is similar to roughly 30 per cent of the other. The biggest area in common is blunt head injury in the context of road accidents. Chapters 2, 3, 4, 7, 19 of this book are similar to chapters 16 - 20 of "Road Safety Theory".

# Contents

# List of Tables and some lists and summaries

# Part A: Blunt injury

Part A has the following chapters.

1. Introduction to this book.
2. Pedestrian impacts: Testing a car's front.
3. Types of blunt impact.
4. Effect of speed (and other variables) on HIC (and other variables).
5. Further consideration of the effects of input variables on output variables.
6. What theory suggests about data description and analysis.

The main part of the book starts with chapter 2, on testing a car's front by projecting an instrumented headform (representing a pedestrian's head) at it. Having given that important specific example, chapter 3 contrasts it with other types of blunt impact. Testing needs to employ some physical measurement that acts as a proxy for the likely injury. Chapters 4 and 5 give some theory about such measurements, and chapter 6 discusses how data obtained in tests should be analysed.

# 1. Introduction to this book

## *1.1 The topic of this book*

Blunt head injury is perhaps the most common fatal or life-threatening injury.

Manufactured objects that may strike someone's head, or that are intended to reduce the severity of injury from an impact, are often tested. From that testing, some measurement is obtained that is intended to serve as a proxy for the injury that might occur. Similar testing is conducted in respect of injury to other parts of the human body. I am dissatisfied with how data from such tests is analysed and reported in research contexts. The aim of this book is to encourage improvement of impact test data analysis. It will do that with the aid of some fairly simple theory about the forces acting on colliding objects. As you might expect, high severity of injury and high likelihood of death are likely to be the consequence of high speed of impact and high stiffness of whatever is struck by the human. But there are a lot of important details, as will be seen in this book.

Theory about injury and damage is likely to mean different things to different people. I had better quickly make clear my meaning, in order not to disappoint the reader. This is not a conventional book on impact injury.

The practical issue motivating this book is the treatment of data in impact experiments. Many people conduct some sort of experiment or test in which one thing (perhaps a substitute for a human) hits another, and the results analysed. For example, an instrumented headform (representing someone's head) might be projected against a car exterior or interior, or dropped on to a surface representing a playground or sports field. Something is measured, such as the acceleration of the headform over the duration (milliseconds) of the impact. But the question of how should the data be presented and what conclusions should be drawn is usually not, in my opinion, answered very well. Analogous experiments are conducted concerned with protective packaging of manufactured goods and gentle handling of agricultural produce.

Let me re-express the previous paragraph in broader terms. The testing of manufactured objects is an important component of the public health system. It is desirable that the best information practicable be obtained from the tests, and that the best use be made of that information. That means understanding the data, and understanding it sufficiently deeply that predictions can be made about what the results would be in different conditions of impact. And that level of understanding means, I think, that a theory is needed.

As I said earlier, many people conduct some sort of impact test.

- It is easy to appreciate that there are lots of contexts in which blunt injury can occur. The way I have organised the examples in chapters 7 - 14 is as follows: (7) transport contexts, (8) sports (with or without helmets and other protection), (9) sports (ground impact), (10) children's play (ground impact), (11) adults, (12) military and security contexts, (13) the Viscous Criterion, and (14) rigid surfaces.
- It is easy to appreciate that packaging of manufactured items is another important context for cushioning. See sections 17.2 - 17.7.
- It is easy to appreciate that fruits need to be handled gently from when they are picked to when they are eaten. See chapter 18.
- But have you thought of the many other specialist situations in which it is important to protect something from damage? Topics that will be considered in the second half of chapter 17 are as follows: damage from hail or windborne debris, frontal damage of cars in crash tests and accidents, aircraft landing gear tests (that were published as long ago as 1931), protection (of roads and other structures) against rock falls, damage to undersea pipelines, impact with a beam, axial impacts with tubes, fibre-reinforced composites, metal honeycomb, impact of bags containing liquid, and shear thickening fluid.

Of course, I know very little about many contexts for injury or damage. But it is worth noting, here in the first section of chapter 1, that quite an elementary method of handling the data from impact tests would greatly add to its value. In sections 7.13 and 20.1, I will say that it seems sensible to plot output variables against input variables on the basis that the relationships may be power funtions. Thus the logarithm of one variable should be plotted against the logarithm of the other. Then it can be asked whether the relationship is approximately a straight line, and what the slope is. And having considered separately each of several pairs of variables and estimated the exponents, the question can then be examined of whether all the exponents are consistent with some particular value of the spring exponent n (the impact surface is regarded as a nonlinear spring).

I should say also that I am thinking of injury or damage from a single severe impact, not from repeated mild impacts. So I am mostly excluding issues with keyboards, shoes, the helmets used in sports in which head impacts are routine, and so on.

There is good evidence from data that the effect of impact speed on the probability of death is rather strong. (I am thinking chiefly of human impacts occurring in road accidents, but the same is probably true of other types of human impact.)

- Even quite a small reduction in impact speed is likely to be worthwhile, therefore.

- It seems likely that high acceleration during impact is more closely linked to death and serious injury than impact speed itself is. Thus even quite a small change to the properties of the surface impacted may also lead to a worthwhile reduction in the probability of death.

The proposition is often put into quantitative form in the following way. The probability of death (p) is a power function of speed at impact (v). That is, $p \propto v^c$, where c is the exponent of the power function and $\propto$ means "is proportional to". Furthermore, c is appreciably bigger than 1, approximately 3. (Usually v should be interpreted as velocity change.) Elementary properties of power functions mean that in the case of small changes in speed, if the impact speed (or, better, velocity change) is reduced by 1 per cent, the probability of death is reduced by c per cent. For more on this, see especially chapter 3 of Hutchinson (2018).

The final point I want to make at this stage is that real people are being killed and seriously injured. I deal with statistics and data on the subject. I write about the subject impersonally. People working in research are accustomed to this style. But I hope that many people outside research will read this book. Some of them have personal experience of impact injury. This book may seem far distant from the suffering they are familiar with. I hope they will not reject this book if that is so.

## *1.2 Outline of the contents of this book*

Part B of this book (chapters 7 - 14) will re-examine published data from impact tests, and try to extract more findings. To make that possible, chapters 3 - 6 of Part A will set out some relevant theory.

I became interested in the subject because head injury is so frequent in road accidents. In particular, the fronts of cars are tested by having a headform (representing a pedestrian's head) projected at them. The headform has an accelerometer in it. The acceleration record over the milliseconds of impact is processed, and summary measures are calculated. More information about this is given in chapter 2, as a specific example of testing of manufactured objects. Having given that important specific example in chapter 2, chapter 3 contrasts it with other types of blunt impact. Part A is completed by the main theoretical chapters (4 and 5), and by implications of theory for data analysis (chapter 6).

Part B (chapters 7 - 14) gives examples from several areas of application. Chapters 7 - 12 deal with transport contexts, sports (with or without helmets and other protection), sports (ground impact), children's play (ground impact), impacts of adults in various miscellaneous contexts, and military and security contexts for impacts. It is understood that the impact tests are relevant to human injury. Most of the examples are concerned with head injury, acceleration being the crucial quantity. Concerning blunt injury of the body (chest and abdomen), the relevant tests are rather different from those of the head, and a quantity called the Viscous Criterion is often calculated. This is not based on accelerations. This is considered in chapter 13. Impacts with rigid surfaces are rather special, again, and relevant data is considered in chapter 14. There is not a special section about injury to animals, but an example relevant to leg injury to cows occurring in the movement of lying down is in section 11.8.4, and finite element modelling of dog paw pads striking rigid ground is discussed in section 14.8.

Part C of this book moves away from blunt injury.

- Chapters 15 and 16 concern penetrating injury.
- Chapter 17 is on impact damage to manufactured items (and especially prevention of damage by use of packaging).
- Chapter 18 is on impact damage to agricultural produce, such as fruits, vegetables, and eggs.

I hope it may be possible for researchers concerned with various types of injury, or with packaging, or with the handling of produce, or indeed with other types of impact, to learn from one another.

Part D concludes the book. The purpose of chapter 19 is to show how generalisation might be made from the specific conditions in which a test result is obtained to the wide range of impact conditions occurring in practice. Chapter 20 sums up with suggestions about data analysis, about theories, about the reasons for theories, about experimentation, about reporting of experiments, and about the results discussed in this book. There are six short appendices.

References are listed alphabetically at the end of the book. References are usually cited because they are relevant to the specific point being made. If, instead, they are mentioned because they are more broadly relevant to the topic or to a related topic, the wording usually makes that clear.

There is some overlap between the contents of this book and "Road Safety Theory", which is also by T. P. Hutchinson and also published in 2018, and is listed in the References as Hutchinson (2018). Roughly 30 per cent of this book is similar to roughly 30 per cent of the other. The biggest area in common is blunt head injury in the context of road accidents. Chapters 2, 3, 4, 7, 19 of this book are similar to chapters 16 - 20 of "Road Safety Theory".

## *1.3 Data and theory in impact biomechanics*

I probably ought to apologise for this book even more than for most of what I write.

But let me try to explain how I see things. What I'm faced with is the following.

- Poor data analysis across a range of applications --- headform impacts, other blunt impacts, penetrating injury, the packaging of manufactured items, the handling of agricultural produce, and so on. What I mean by poor data analysis is that the quantity of data is low, the analysis is often unsystematic and disorganised, and theory is not used to shape the process or the conclusions.
- Even in isolation, away from demands of real experiments and data, there is an apparent lack of interest in theory. What I mean by theory is a connexion between experimental inputs and experimental outputs that is derived from the forces acting in the impact. (Experimental inputs are such things as impact speed; experimental outputs are such things as proxies for severity of injury; the forces acting might be functions of instantaneous displacement and instantaneous speed.)

I am rather surprised by this situation. It seems to me that I could make a dozen errors per chapter, and still advance the subject a step or two.

That is why, even though I don't know much about either medicine or engineering, I have written a book on impacts, biomechanics, and injury. Well, perhaps I should not say that, I should say instead that the book is about the data that arise in experiments relevant to protecting people in impacts.

I am concentrating on very broad-brush theories and methods of data analysis. I do not deny the value of much more detailed or specific approaches.

## *1.4 How much testing should there be?*

From what I have read, do I think there is about the right amount of testing conducted, or too much, or too little?

I'm not a practical experimenter. I hesitate to attempt an answer. My chief complaint is with what I perceive as a failure to integrate experimentation, theory, and data analysis. I think this wastes a lot of the potential benefit to immediate users and to the wider public.

But, having said that, my impression is that there ought to be about 100 times more testing than there is. (I am referring to research-oriented testing.)

- Practically every study I've looked at ought to have done 10 times more tests, just to achieve a halfway decent response to its narrow aims.

- And the number ought to be multiplied by 10 again in order to begin to integrate the narrow aims of a particular study with theory and with wider experimental experience.

Am I crazy? Wouldn't that be very expensive?

- Firstly, I am doing what I can, by writing this book, to get theory widely known that will help get the most out of test results, and economise on the number of tests required.
- Secondly, the problems of concern are big ones. Road accidents account for 1 or 2 million lives lost per year worldwide. Plus, people get killed and injured by blunt impact in various other ways. Packaging of manufactured goods so as to avoid damage is an enormous industry. Handling of agricultural produce so as to avoid damage is an enormous industry.

So, no, I do not think my opinion is an unreasonable one.

It is reasonable to say that the expense of testing has limited the amount of testing carried out. I suggest that may not be the full story. Another reason may have been a lack of incentive. Methods of data analysis that have implications for theory may not have been widely known. This book describes such methods --- in chapter 6, and with many examples in later chapters. Consequently, experiments can be conducted with confidence that it will be possible to condense the data into an intelligible narrative that suggests conclusions about the force acting during the impact.

## *1.5 What's the use of theory?*

You may think that the purpose of a theory is to tell us the answer if we do not have direct empirical knowledge of a question. I do not positively disagree with this, but in my opinion it is too optimistic.

The theories I have experience with are quite poor at doing that. There is a lot wrong with them. Instead, they help the researcher in less definite but nevertheless real ways --- in organising data and in organising thinking, for example. See section 20.3.

I said in section 1.1 that this is not a conventional book on impact injury. There are some very good ones. There is a vast amount in them that you will not find here. I encourage you to consult them.

> However, quite a popular approach in many fields of science is to have in mind a small number of dependent variables of interest, and to ask how these are affected by a small number of independent variables. Progress is then attempted using theory and using analysis of experimental data. That is the sort of thing I am attempting in this book. I do not think the conventional literature does that very well.

## *1.6 The technical level of this book*

This book will use mathematics --- mathematical symbols and notation, equations, and algebra. This will include some calculus --- differentiation, integration, and differential equations. I will do my best to explain in words the core meaning of the most important equations.

A lot of this book is not difficult, and I think an undergraduate in physical sciences, mathematics, engineering, or medicine ought to be able to understand everything here.

> The central idea in chapters 7 - 14 is that when examining data from impact tests (for example, something is dropped on to something else, and the acceleration pulse is measured with an accelerometer), it is often sensible to plot the logarithm of the dependent variable (e.g., maximum acceleration) versus the logarithm of the independent variable (e.g., impact speed).

Many school students are capable of doing that, and certainly undergraduates can. See especially sections 6.1, 6.3.1, and 20.1 for the treatment of data.

I hope that many readers of this book will come from outside the world of safety research. For some, decades may have passed since they used mathematics in a formal way. It may be a difficult book for them. My advice can only be based on my experience when reading about something unfamiliar, as I am not an expert on reading. Part of that advice is to press on at a reasonable rate. One can get something out of text from only partial understanding, and to a degree appreciate the facts being marshalled, the tools being used in the argument, the conclusions reached, and how sound are those conclusions. Another part is that you should not be surprised if you sometimes spend 30 minutes reading a single page. That is what it takes when you are at the borderline of understanding --- but only do this if you really are gaining something, and it is better to move on if you find yourself staring at a paragraph without real engagement of your mind.

## *1.7 Words and explanations*

*Displacement, deformation, distance*. These words all refer to how far something moves after initial contact with something else. Unsupported metal in the middle of a car's bonnet deforms a substantial distance (several centimetres) when struck by a pedestrian's head or shoulder. Injury is likely to be much less than if the pedestrian had hit the car's A pillar, which is very stiff and deforms very little. In studies with instrumented headforms, displacement is usually not measured directly, but by double integration of acceleration. (The A pillars of a car are at the left and right of the windscreen, that is, forward of the front doors.)

*Translational* vs. *rotational* movement. In physics, these terms are used in contrast: translation means movement of location.

*Velocity*. This term is sometimes used in a way such that it is positive in one direction and negative in the opposite direction. But sometimes it is used quite casually to mean speed. (See also section 1.9.)

*DeltaV*. This means change of velocity.

*Acceleration and deceleration*. As is commonly done, I will sometimes use acceleration in distinction to deceleration, and I will sometimes use acceleration in a wider sense to include deceleration. Acceleration is often expressed in units of g, the acceleration due to gravity, which is approximately 9.81 m/sec/sec. An impact of 20 g, for example, refers to approximately 196 m/sec/sec.

*Stress and strain*. Stress is force per unit area. Strain is amount of deformation, usually expressed as a proportion of the original measurement.

*Normal*. (a) In the context of geometry, this often means at a right angle (90 degrees, perpendicular). An impact test may be set up so that a sphere falling vertically strikes a flat horizontal plate: the impact is normal. (b) In the context of statistics, a normal distribution is a commonly-used continuous probability distribution.

*Coefficient of restitution*. Referring to an impact of two bodies, the coefficient of restitution is the absolute value of the ratio of final relative velocity to initial relative velocity. In the types of impact in this book, the coefficient of restitution is usually small, and can sometimes be thought of as zero.

*Primary safety and secondary safety*. In road safety, primary safety refers to the avoidance of crashes, secondary safety refers to reduction of injury given that a crash has occurred. Secondary safety measures include seat belts and air bags for vehicle occupants, and helmets. For pedestrians, a car's bonnet should act as a cushion, and improving it in this respect is another important secondary safety measure.

*Headform*. This approximates the dimensions and weight of the human head. It is instrumented with accelerometers. The accelerations during the milliseconds of impact are recorded and processed.

*Hybrid III*. This is a well-known crash test dummy.

*Bottoming out*. Consider a hard object hitting a cushion. The hard object might be a pedestrian's head, or might be a pedestrian headform used in impact tests as described in chapter 2. The cushion might be a car's bonnet. At low speeds, the cushion will ensure that the hard object will be stopped fairly gently. But that is only the case if the cushion is sufficiently thick. If the cushion is thin, or if the impact speed is high, the whole depth of the cushion will be used up. In the case of a pedestrian head hitting a bonnet, the bonnet may deform so much that it strikes very stiff structures in the engine compartment. Consequently, for the head, there is great increase of the severity of the impact. This is referred to as bottoming out. In that case there is a sudden change in stiffness. The change may be a little less sudden in the case of foam that covers metal. The foam may behave linearly up to a substantial fraction of its depth, before gradually increasing in stiffness and then sharply increasing in stiffness. See also Appendix 1.

*Severity of injury*. Data about road accidents and road casualties that is routinely collected by the police often includes a classification of severity of injury. For example, the categories used may be fatal, serious, slight, no injury. When these terms are applied to accidents, they refer to the most seriously injured person.

- Many methods of classification are used by various police forces around the world. For example, it is common for fatal injury to include deaths at the scene of the accident or within 30 days, and to exclude later deaths. It is common for serious injury to refer to injury requiring admission to hospital, or a broken bone. (The classification may be made by someone who is not a medical expert, and who is using imperfect information.)
- When reference is made to accidents or to people involved, the term "serious" may be used for brevity, and actually mean serious or worse (that is, the fatalities are included).
- The term "severity", in the context of a set of accidents or people involved rather than for an individual, is likely to refer to a proportion, such as the proportion who are killed or seriously injured.

*Ballistic*. This refers to bullets, fragments, and so on.

*General abbreviations*. i.e. = that is, e.g. = for example, et al. = and others (this is used to avoid a long list of joint authors of a particular work), vs. = versus, meaning in contrast to, and also used in referring to the two axes of a graph or scatterplot (as in y vs. x).

## *1.8 Some terms used in mathematics, statistics, and data analysis*

It is not practicable to give a short course in algebra, calculus, and statistics in this book. But I should explain a few terms.

*Symbol for multiplication.* Both × and the dot . are used as symbols for multiplication.

*Power function, and exponent.* When some number x is multiplied by itself, x × x, this is written as $x^2$. Similarly, x × x × x is written $x^3$. The expression $x^c$ is termed a power function of x, and c is called the exponent.

- Product of powers of x: $x^b.x^c = x^{b+c}$
- Successive raising to power: $(x^b)^c = x^{b.c}$

I occasionally use the symbol ^ to mean "raised to the power". Thus x^c and x^(c) mean $x^c$.

*Symbol for proportionality.* $\propto$ means "is proportional to".

*Symbol for differentiation with respect to time.* If x is distance, the rate of change of distance (speed, the first derivative of distance with respect to time) may be written as x', and acceleration (the rate of change of speed, that is, the second derivative of distance with respect to time) may be written as x".

*Logarithm.* It is common to use natural logarithms rather than logarithms to base 10; ln is the abbreviation used for natural logarithm (logarithm to the base e, where e is Euler's number).

*Brackets.* Brackets are used for two purposes: to group quantities together, and to denote the argument of a function.

*Independent variable and dependent variable.* When calculating one thing from one or two or more other quantities, the result might be known as the output or the dependent variable. The quantities from which it was calculated are the inputs or independent variables. (Independent variable is rather a poor name in the sense that one independent variable may not actually be statistically independent of others.)

*Median.* The median is a sort of average. If numbers (observed data) are arranged in order of magnitude, the median is the middle one. Compared with the mean, the median has some disadvantages. It also has some advantages: it is meaningful when the numbers are only ordinal, not fully quantitative; and it is less sensitive to observations that are unusually small or large.

*Ordinal data*. This refers to numbers, or other things, that can be put in order, but which it is meaningless to add or subtract. Injury severity is an important example: one method of classification might be as fatal, serious, slight, none, and another might be as 6 (maximum, virtually unsurvivable), 5 (critical), 4, 3, 2, 1 (minor), 0 (no injury).

## *1.9 Notation*

Please be aware that notation (what symbols mean) is not the same throughout this book. A symbol such as x may be used to mean one thing in one chapter and another in another chapter.

There are some quantities (e.g., velocity) that are vectors. These are positive in one direction and negative in the opposite direction. I am sometimes careful about this, and sometimes not. What I mean is, suppose there is an impact of a vehicle that was originally travelling at 50 km/h, and after impact travels at 10 km/h in the opposite direction.

- I might say change of velocity is 50 - (-10) = 60 km/h. (Here I am being careful to represent a change as a difference, and to represent the opposite direction by the opposite sign.)
- Or I might say change of velocity is 50 + 10 = 60 km/h. (Here I am writing casually and presuming that the description of the event is sufficient to make the change clear.)

## *1.10 Credibility of data*

Data is useful. But you should not put it under pressure that is too much for it. When you examine data carefully, you often find something is wrong with it. That is an important reason why theory is needed, to help us perceive the correct message in imperfect data.

# 2. Pedestrian impacts: Testing a car's front

## *2.1 Introduction*

This chapter will discuss the principles for minimising the danger posed by a car's front (especially the bonnet) to pedestrians and other vulnerable road users. (Vulnerable road users is a phrase used to include pedestrians, cyclists, motorcyclists, and others outside a vehicle.) This example is important in itself, and to serve as a base from which other types of impact can be considered. Much of this chapter is based on part of Hutchinson et al. (2011). Chapter 16 of Hutchinson (2018) is similar to this chapter. A highly relevant book is that by Simms and Wood (2009).

In recent decades, increasing attention has been paid to improving car frontal design in order to minimise pedestrian injury. The first point to make is that, for pedestrians and other vulnerable road users, the exterior of the car can and should be designed to act as a cushion to protect them from stiffer structures underneath.

Head injuries are a common cause of death in pedestrians. They are usually from vehicle contact rather than ground contact. The fronts of cars are low enough that, except in the case of very young children, the pedestrian's head is not struck by the part of the vehicle that is near-vertical above the bumper, but the pedestrian's body rotates towards the bonnet. The head is then struck by either the bonnet or the windscreen of the vehicle.

When the head is struck, it is accelerated by the impact. The mass being accelerated is approximately that of the head, but to some extent modified by the rest of the human body. This mass is referred to as the effective mass.

Part of the effort towards frontal design improvement involves projecting a free-flight instrumented headform against a number of locations on the exterior of the car and obtaining a record of its acceleration over the milliseconds of the impact. Such tests are conducted at a specified speed (11.1 m/sec, which is 40 km/h), and with headforms of specified mass (3.5 kg and 4.5 kg) and dimensions. Changes to specifications of the conduct of tests have occurred over the years, and are likely to continue. The acceleration trace is summarised by calculating the HIC (Head Injury Criterion). This is believed to reflect likely injury severity. (For how it is calculated, see section 4.2.1.) In other contexts, maximum (peak) acceleration is used for a similar purpose. HIC and maximum acceleration might be referred to as proxies for injury severity, or as injury response functions.

Before the head is struck, it is common for the lower leg and the upper leg to be struck. Correspondingly, there are tests of other locations on car exteriors using free-flight legforms. Head injury is more common as a cause of death, and so receives more attention.

The human head and the instrumented headform are regarded as rigid in comparison with the car bonnet: they do not deform, the bonnet does. The human head and the instrumented headform are regarded as small in comparison with the car bonnet: they accelerate, the bonnet does not. Injury is regarded as a consequence of the acceleration. HIC is regarded as a reasonable method of summarising the acceleration trace for this purpose.

Several points about bonnet stiffness are worth making straightaway.

- It might be thought that the less stiff, the better. But that is true only up to a point. The bonnet is protecting the pedestrian's head from contact with very stiff structures in the engine compartment of the car. It needs to be stiff enough to do that.
- For a given speed, a good approximation to the optimal stiffness would be that for which the clearance distance (the space under the bonnet before stiff structures are reached) is exactly used up in stopping the headform. It might be better for stiffness to be slightly less than this, as even stiff structures are unlikely to be very injurious if the residual speed when they are reached is low.
- But that stiffness will not be optimal for lower and higher speeds. At lower speeds the stiffness will be too great, and at higher speeds the stiffness will be too low.
- Consequently, it would be desirable for results to be obtained for a range of realistic speeds. Testing is a potential means of obtaining those results, but sometimes a simple calculation may be sufficient. A similar conclusion applies to having a range of headform masses.

See also sections 2.3 and 19.4.1 and the further discussion in Appendix 6.

## *2.2 General principles of bonnet design for pedestrian safety*

Partly as a result of impact testing, some general principles of bonnet design are now well understood.

- Projections and sharp corners and edges should be eliminated.
- There should be plenty of clearance distance between the underside of the bonnet and very stiff structures such as the engine and the suspension towers.
- One strategy for achieving extra clearance distance is to use a pop-up system, that quickly lifts the rear edge of the bonnet when activated. For a review of market penetration and safety performance of such systems, see Ames and Martin (2015).

- The bonnet should be yielding, but not so much so that it deforms too easily and fails to prevent the pedestrian's head striking a very stiff structure. This dilemma requires some intermediate, optimal, degree of stiffness to be found.
- The very stiff structures underneath the bonnet should be made less stiff, or frangible.
- The coefficient of restitution (see section 1.7) for the pedestrian-vehicle contact should be low. The pedestrian should tend to stick to the bonnet; bouncing is more dangerous.
- If it is practicable to exercise some control over the shape of the acceleration pulse, the peak of this should be early rather than late in the impact. That is, the bonnet material should be damped, i.e., be stiffer early in the impact (when speed is high and bonnet deflection is low) than later. The importance of high accelerations rather than low in causing injury might be thought to imply that for a given velocity change, the acceleration should be constant over the time the pulse lasts. However, high acceleration occurring early also disproportionately reduces the distance travelled. Thus to minimise HIC under the constraint of a given available clearance distance, acceleration should be higher early in the impact (Okamoto et al., 1994). In the context of helmet linings, Cheng et al. (1999, p. 306) mention breakaway materials as a possible method of achieving a high force early.

The desirability of eliminating anything sharp or projecting, and of having a low coefficient of restitution, were appreciated by Wakeland (1962). Some years later, the account in Harris (1976) is a considerable advance, with a recommendation that "Hidden components should be terminated well below bonnet level to allow depth for deformation. Examples are the engine and fittings, front suspension and the side walls of the engine compartment." At about the same period, McLean et al. (1979, pp. 39, 42) drew attention to this issue from the perspective of pedestrian injury cases that had been investigated in Adelaide in 1976-1977.

These principles indicate that regulations and recommendations could attempt to control separately several aspects of bonnet design, instead of the global performance summary represented by the Head Injury Criterion --- surface sharpness, clearances, bonnet stiffness, stiffness of under-bonnet structure, coefficient of restitution, and damping of the bonnet. However, car companies have a great deal of expertise, and it seems reasonable to focus on overall performance and leave the method of achieving that to the vehicle designer. There may be other contexts of blunt head injury, though, in which it would be appropriate for regulations or specifications to refer directly to analogous aspects.

Routine headform testing has been carried out by the European New Car Assessment Programme (Euro NCAP) and the Australasian New Car

Assessment Program (ANCAP) for some time. The tests are conducted for consumer information purposes only. That is, this is not regulatory testing, and poor performance will not stop a vehicle from being sold.

Lawrence et al. (2006) demonstrated several methods for improving the pedestrian test performance of two cars: a Ford Mondeo and a Landrover Freelander. These vehicles were compared with the better-performing Honda Civic. Several design improvements were suggested, most of which involved increasing clearances and reducing the stiffness of bonnet supports. Another feature of the Honda Civic was that the stiffer structures beneath the bonnet were designed to break away --- for example, the windscreen wiper motor and the brake fluid reservoir. The features of this study illustrate the progress that was being made by one manufacturer (Honda) at the time, in contrast to manufacturers that had not considered pedestrian safety as a high priority. Other references about improving the pedestrian safety of cars include Clemo and Davies (1998), Han and Lee (2003), Hobbs et al. (1985), Kuehnel and Appel (1978), Wollert et al. (1983), and Yoshida et al. (1999).

Since 1997, the impact laboratory at the Centre for Automotive Safety Research, University of Adelaide, has conducted pedestrian headform and legform testing on behalf of ANCAP, plus tests for other clients and other purposes. Ponte et al. (2013) describe this activity. See Appendix 2 of the present book for improvements that have been noted in respect of head impacts. For the Euro NCAP test method (and, in particular, the changes to headform impact procedures from 2013), see Zander et al. (2015). See Whiteside (2010) for information about seven pedestrian headform protocols using a 2.5 kg headform at 11.1 m/sec or a 3.5 kg headform at 9.7 m/sec, and seven protocols using a 4.5 kg headform at 9.7 m/sec or a 4.8 kg headform at either 9.7 or 11.1 m/sec.

There is likely to be further progress in coming years as the designs of bonnets and other relevant vehicle structures are revised. In many respects, testing protocols for regulatory and consumer information purposes will be expected to work well. For stiffness of hard structures, clearance under bonnet, and coefficient of restitution, it is appropriate for regulation to encourage design in one direction: the softer that hard structures are, the better; the greater the clearance distance, the better; and the lower the coefficient of restitution, the better. Bonnet stiffness, however, is a special case, and this will be discussed below.

## *2.3 Bonnet stiffness as a special case*

In the case of bonnet stiffness, there is an optimum: too stiff, and the bonnet is injurious; not stiff enough, and the pedestrian’s head bottoms out, that is, strikes the very stiff structures in the engine compartment. The optimum stiffness succeeds in bringing the head to rest just as the very stiff structures are contacted; that is, all the clearance distance is used up. (This description is an approximation for several reasons, including that contact at some very low speed may not be greatly harmful, that stiffness may vary with deformation distance, and that stiffness may depend on speed as well as on deformation.)

However, the stiffness that is optimal at one speed will not be optimal for other speeds.

- In particular, severity of injury at higher speeds may be very bad because of bottoming out --- especially if the bonnet is optimised for quite low speed impacts, i.e., is fairly soft.
- Severity of injury at speeds lower than that for which stiffness was optimised will also be worse than necessary, as all the available clearance distance is not used.

Consider severity of injury as a function of speed of impact, with some particular clearance distance being available before bottoming out occurs. (I will not refer to any specific definition of severity, as I intend to give a valid general picture whatever definition is used.)

- Suppose the bonnet to be optimised for an impact at 40 km/h, say. At speeds of impact lower than 40 km/h, there is gradually increasing severity of injury with increasing speed, as more and more of the clearance distance is used up. At higher speeds of impact, there is sharply increasing severity of injury, as bottoming out gets worse and worse.
- Suppose the bonnet to be optimised for an impact at 50 km/h. At speeds of impact lower than 50 km/h, there is gradually increasing severity of injury with increasing speed, as more and more of the clearance distance is used up. At higher speeds of impact, there is sharply increasing severity of injury, as bottoming out gets worse and worse.
- And the following two comparative statements are fairly evident. At speeds lower than 40 km/h, severity for a bonnet optimised for 50 km/h is higher than for a bonnet optimised for 40 km/h. At speeds higher than 50 km/h, severity for a bonnet optimised for 50 km/h is lower than for a bonnet optimised for 40 km/h.
- At some speed a little over 40 km/h, the lines for the two bonnets of different stiffnesses cross over.
- At low speeds, the bonnet optimised for the higher speed performs worse: it is too stiff.
- At high speeds, the bonnet optimised for the higher speed performs better: it absorbs more energy before bottoming out occurs.

A stickler for accuracy may object that there is no way of measuring injury severity on a quantitative scale, that injury severity should be considered an ordinal variable, and that consequently it is meaningless to refer to an increase of injury severity as being gradual or sharp. I mostly agree, but the wording above is sufficient for present purposes.

I would expect the above to be approximately true quite generally. However, in any specific case there are likely to be many important details. For example, materials having nonlinear stiffness or velocity-dependent stiffness may be available, though perhaps at increased cost.

According to the argument above, the line representing dependence of injury severity on impact speed for a bonnet of one stiffness may cross over that for a bonnet of another stiffness. The reason is that the bonnet of lower stiffness bottoms out at a lower speed.

- If such cross over is observed in empirical data, it may be due to bottoming out, but that is not the only possibility.
- Instead, the laws governing impact behaviour may be different for the two bonnets.
- A class of possible laws will be proposed in section 4.2.5; if (for example) the exponent n in that class of laws is different for different bonnets, cross over may occur.

See Appendix 6 for a suggestion about optimal stiffness when there are several speeds of impact (and therefore there needs to be some sort of process of averaging).

I understand the consequences of test speed have been controversial in the testing of motorcycle helmets. Consider a set of helmets that have passed a compulsory test at a relatively low speed, and another set of helmets that have passed both the compulsory low-speed test and an optional test at a relatively high speed.

- My opinion is that it is reasonable to be concerned that good performance at high speed may have been achieved at the expense of poorer performance at low speed.
- Becker et al. (2015) provide evidence that this has not in practice happened. They reported results of tests at several speeds of helmets from two sets as described. Average performance of the second set was better at high speed than that of the first set, and was similar at low speed.

## *2.4 Discussion*

Testing using a single test speed and headform mass will lead to improved pedestrian safety in those conditions. Several types of design change are available (see section 2.2 and Appendix 2). However, an improvement achieved by making the bonnet less stiff may worsen safety at higher speeds and higher masses because of bottoming out, and an improvement achieved by stiffening the bonnet may worsen safety at lower speeds and lower masses. Thus to achieve balance between these conflicts, estimates of safety performance at other speeds and headform masses are required. Furthermore, it will be desirable to develop methods of calculating an average level of performance that take into account the frequency with which different speeds and head masses occur.

The desirability of obtaining results over a range of speeds does not necessarily imply that the number of tests would be drastically increased. Firstly, when clearance distances are sufficient that bottoming out does not occur, simple calculation (based on relationships in section 4.5.1) can convert the Head Injury Criterion obtained at one speed and headform mass to other test conditions. Secondly, testing could be carried out for only some of the combinations of conditions. (The combinations might be randomly chosen, as the aim would be to get a good estimate of the safety performance of the vehicle, rather than to gain new knowledge about some location of impact on it.)

In general terms, there is a steep increase of severity with speed of striking a stiff structure beneath the bonnet. Thus the chief penalty for failing to take account of the range of real-world speeds and head masses is likely to be at high speeds and high masses. However, this is poorly understood quantitatively, and both experiment and modelling are needed to improve knowledge of what designs and materials are optimal.

The principles given here apply to other large structures that the head may impact --- both in road safety contexts (e.g., the car interior), and in quite different contexts (e.g., sport, playgrounds, military). In particular, there will always be a degree of concern that the choice of particular conditions (e.g., impact speed) in which to test will lead to a particular choice of stiffness, which will be too low to prevent bottoming out in more severe impact conditions (higher speed, greater effective mass).

# 3. Types of blunt impact

## *3.1 Introduction*

An important theme in this book is the importance of testing, and specifically, the importance of impact testing. It is highly desirable that a test result should have some implications beyond the specific conditions of the test. For example, it would be useful to have a formula for what HIC might be at some speed other than the impact speed employed in a pedestrian headform test. Results of that type are given in Hutchinson (2013) and in chapter 4 below. The question naturally arises of whether those results are useful in other types of impact testing. The purpose of this chapter is to contrast the pedestrian headform tests described in chapter 2 with other forms of impact testing. The headform tests will be put into the context of four simple types of test. Chapter 17 of Hutchinson (2018) is a shorter version of this chapter.

In designing a test, one would like to know what physical quantity is most closely responsible for injury. Possibilities include acceleration, force, and deformation. Although this chapter will not answer this very difficult question, a number of important issues will be identified and clarified, even if a gap remains between physical variables and biological effects. This chapter does at least serve as a warning that results that might be obtained for the pedestrian headform tests of chapter 2 (for example, in chapter 4) might not be transferable to other types of impact test.

Someone interested in human injury may conclude that the reason for uncertainty about what really causes injury is the difficulty of experimenting with live humans, and thus measuring biological effects. However, I think that may not be the whole story, as I have seen indications in the literature on impact damage of manufactured objects that some researchers are not confident they are measuring the most appropriate quantities. The title of one paper begins "Is the maximum acceleration an adequate criterion" (Suhir, 1997).

The considerations here are in a sense elementary. Nevertheless, they are probably unfamiliar to many people. Even specialists in one field of application (e.g., pedestrian head injury) may not know much about another (e.g., chest injury from a punch).

- Only blunt (non-penetrating) injury is considered (penetrating injury from bullets and fragments will be discussed in chapters 15 and 16).
- Only injury from translational (not rotational) movement is considered.
- The geometry of the impact is assumed to be the simplest, as when an object is dropped on to a flat surface.

## *3.2 Classification of types of impact*

My opinion is that a helpful way of comparing different types of impact is as follows. Consider the two objects that collide. There are four binary contrasts between them.

1. One is human, the other is inanimate.
2. One is moving, the other is stationary. (Although one is stationary, it is free to move, not clamped in position.)
3. One is large, the other is small. I am using these terms to mean that acceleration of the large or massive object is negligible, and thus all the acceleration is of the small object. If a small object is clamped in position, rather than being free to move, it must be considered to be the large object.
4. One is rigid, the other is deformable. I am using these terms to mean that deformation of the rigid or stiff object is negligible, and all the deformation is of the deformable or yielding object.

The first two of these contrasts require only brief comment.

- As to the first, attention is concentrated on what happens to the human. Or, more generally, attention is concentrated on the object that might be damaged. This is usually a human. (The packaging of manufactured goods and the handling of agricultural produce are also important areas of application, and will be discussed in chapters 17 and 18.)
- As to the second, what matters is the relative velocity of the impacting objects, not the identification of which is moving and which is stationary.

The other two contrasts --- in respect of mass, and in respect of deformability --- will be considered further in this chapter. They imply four types of impact that need to be distinguished.

- Small rigid human, impact with large deformable object. Pedestrian headform tests are examples of this type, see chapter 2.
- Large deformable human, impact with small rigid object. Example: human chest is struck by a hard ball.
- Small deformable human, impact with large rigid object. Examples: human chest is punched by a large robot, human head strikes concrete floor; and fruits and vegetables striking the sides and bottoms of containers and channels are likely to be analogous.
- Large rigid human, impact with small deformable object. Example: human head is struck by a plastic bullet.

This list is given largely in order to discuss injury from acceleration as contrasted with injury from deformation, and implications concerning proxies for injury. Intermediate cases, in which the two objects are of comparable mass or comparable stiffness, are more complicated and are mostly outside the scope of the present discussion; there will be some discussion of hard sports balls and so-called less lethal munitions, which have much less mass than a human head (for example), but are fast enough to sometimes cause injury.

## *3.3 Proxies for injury*

Injury response functions, and injury metrics, are other terms for proxies for injury.

An impact test needs to measure something that reflects what injury severity would probably be. In pedestrian headform impact tests (chapter 2), the headform accelerates a lot and deforms to a negligible extent, and the proxy for injury severity is based on acceleration. That sounds simple enough, but there are complications. This section will discuss some of them.

Injury severity itself is a rather vague and ambiguous concept. Most measures of injury severity are largely measures of threat to life; and they are ordinal, not truly quantitative.

### 3.3.1 A contrast between acceleration and deformation

A number of different proxies for injury are in use. In each of the types of impact described in section 3.2, only one thing deforms, and only one accelerates. The softer thing (e.g., the bonnet, not the headform) deforms, and the smaller thing (e.g., the headform, not the bonnet) accelerates.

- Some of the proxies for injury are based on acceleration (e.g., maximum acceleration, and HIC). These are only relevant when the human accelerates, that is, when the human is the small object in collision with a large object.
- Other proxies for injury are based on deformation of a human (e.g., maximum deformation, and maximum Viscous Criterion). These are only relevant when the human deforms, that is, when the human is the deformable object in collision with a rigid object.

Thus it appears that if the human is large and rigid (e.g., impact of a deformable plastic bullet with a head), most of the common proxies for injury are unsuitable. Maximum force may be suitable.

### 3.3.2 Use of acceleration-based proxies for injury in test protocols

The use of acceleration-based proxies for injury, such as HIC and $A_{max}$, is often specified in test protocols. Does this constitute evidence that the experts who wrote the test protocols believed that acceleration (rather than force or deformation or something else) is the most important factor in injury causation?

For some discussion of this question, see Appendix 3. I think the answer is probably no. The experts may be of the opinion that what matters is something other than acceleration (e.g., deformation of the head), but when comparing vehicles, this other quantity correlates highly with headform acceleration. (Section 5.2.2 will indeed obtain such a relationship.) The present chapter is making the point that some types of injury may be related to acceleration of the human and other types of injury may be related to deformation of the human. Hence the possibility must be mentioned that acceleration is not important in itself but as an indicator of human deformation.

### 3.3.3 Different types of injury, and different causes of death

An important complicating factor is that there are lots of different types of head injury that cause death. Furthermore, it is quite likely that the mechanical change to the head that was responsible for the injury is different for the different types of injury. I have in mind the distinction between soft tissue injuries (e.g., of the brain) and bone injuries (e.g., of the skull). An expert might believe (for example) that compression is the real cause of one type of injury, and force is the real cause of another type of injury.

- Monea et al. (2014) made a distinction between (a) injuries localised to the point of impact of the skull and underlying brain, and (b) distributed injuries, meaning those elsewhere in the brain. They consider that the second group are associated with rotation, with some due to tangential inertial force related to rotational acceleration, and others due to centrifugal inertial force related to rotational velocity.
- In a finite element study, Oikawa et al. (2017) assessed different head injuries by different variables: skull fracture by skull strain, intracerebral hematoma by brain pressure, brain contusion by brain negative pressure and von Mises stress, and diffuse axonal injury by von Mises stress.
- In another finite element study, Tse et al. (2017) assessed skull fracture by von Mises stress, and brain injury by intracranial pressure, principal strain, and shear strain.

- Use of a specific proxy for a specific type of injury suggests the authors considered that specific proxy suitable in their study for indicating occurrence of that specific type of injury. However, it does not unambiguously imply the authors considered that specific proxy is the "real cause", as I have called it, of that specific type of injury.

A particular reason for concern with these matters is that it is highly desirable to be able to calculate results that would be obtained in other conditions from the results obtained in specific test conditions. That calculation may give different results depending on what proxy for injury has been adopted. Possibly also a headform, or a finite element model, might be biofidelic in some respects (e.g., mass) but not others (e.g., stiffness). Nsiampa et al. (2012) seem to agree.

We might conclude that an exact answer is out of reach. We have to decide on something to measure in tests, but it should be remembered that in respect of what the real cause of injury is, the decision might be wrong or inaccurate. Ommaya et al. (1994, p. 546) wrote as follows: "Peak and average linear and angular accelerations, including their duration and rate of onset, intracranial pressure, volume changes of the skull, and force and energy applied to the cranium have all been used .... but it is not yet conclusively proven that any of these parameters are optimal for trauma correlation."

### 3.3.4 Discussion

The intention of this chapter is not to say definitely that one type of impact needs one class of proxy for injury, and another type needs another class. It is more cautious than that. Probably the key point is that in unfamiliar contexts, one should question whether an acceleration-based proxy for injury (such as HIC or $A_{max}$) is appropriate, or whether deformation might be more important, or something else.

The question of what matters in regard to the severity of head injury has other dimensions to it. Translational motion or rotational motion? Force rather than acceleration? How important is duration? Brain injury or skull fracture? Localised or diffuse? How important are location on the head and direction of the impact? These questions have to be left aside. See, for example, Antona-Makoshi et al. (2016).

## *3.4 Large deformable human*

The object that is large may be deformable. If this is inanimate, the situation is the original one exemplified by a headform hitting a car's bonnet.

If the large deformable object is human, the situation is similar to a missile (such as a hard ball) striking the thorax of a person, the deformation being of the thorax.

The physical proxy for injury will be different from the headform case. When the human deforms, popular proxies for injury include maximum deformation of the human and the Viscous Criterion ($VC_{max}$, proportional to the maximum of the product of deformation and rate of deformation of the human).

## *3.5 Large rigid human*

The common feature of the third and fourth types of impact is that the large object is the rigid one, and the small object is the deformable one.

Consider a small soft missile (such as a deformable plastic bullet) striking the head of a person. If its speed is high, it may be dangerous despite being small and soft. (Plastic bullets are not intended to be fired at the head, but unintended impacts may happen.) Both acceleration and deformation refer to the inanimate object, not to the human. Thus many proxies for injury that are used in other situations appear to be unsuitable.

Maximum force may be suitable. Oukara et al. (2013, 2014) view intracranial pressure as being a good indicator of damage to the head, and maximum force as being a good substitute for intracranial pressure. Pearce and Young (2014) share that attitude to some extent, but with the important reservation that additional phenomena occur if impact duration is very short (less than about 3 msec, and especially if less than 1 msec).

However, acceleration-based proxies may be relevant to the important problems of hard sports balls or less-lethal munitions striking the head: even though the velocity change of the human is small, acceleration of the head may be sufficient to be injurious. McIntosh and Janda (2003) measured headform accelerations, and they were quite high.

Furthermore, rejection of proxies based on acceleration or deformation may be premature. It may be possible to use these in the context of a theory that estimates the acceleration of both objects and the deformation of both.

## *3.6 Small deformable human*

Examples of impacts in which the human is considered small and deformable include a human head striking a concrete floor, and a human chest being punched by a large robot. Damage to fruits and vegetables striking the sides and bottoms of containers and channels is likely to be analogous. Because the human is small, the proxies for injury that are based on acceleration may be suitable for use. Because the human deforms, the proxies for injury that are based on deformation may be suitable for use.

Maximum acceleration and HIC can be measured with a headform, they are used in the case of the small rigid human, and are likely to be used in the case of the human head being small and deformable. There are two particular reasons for concern, however.

- Firstly, results for an unhelmeted headform falling on to concrete or other rigid surface may not be valid: what happens to an unhelmeted head can only be estimated if the headform has the same deformation characteristics as those of a human head.
- Secondly, the question arises of whether HIC and maximum acceleration are good proxies for injury. They are based on acceleration at the centre of gravity of the head. Presumably this is thought of as likely to reflect the general level of head injury, without being exact about the location or the mechanism, and probably diffuse brain injury is thought of as one of the important forms; no attempt is made to consider localised injury at the impact location, or to convert acceleration to force. (Examples of studies that measured local load distribution are those of Ouckama and Pearsall, 2012, who used an array of sensors 2 cm apart, and Ouckama and Pearsall, 2014.)

For impacts with rigid surfaces, see chapter 14.

## *3.7 Comments on more complicated contexts*

Small rigid human (vs. large deformable object).
Large deformable human (vs. small rigid object).
Large rigid human (vs. small deformable object).
Small deformable human (vs. large rigid object).
Having made that list of types of blunt impact, I am now going to somewhat back away from it.

More complicated contexts, in which the two masses or the two stiffnesses are comparable in magnitude, may be encountered. And I think they might be more commonly encountered than the large rigid human and the small deformable human of sections 3.5 and 3.6.

The three most important types of impact would then be as follows.

- Small rigid human, implying an acceleration-based proxy for injury.
- Large deformable human, implying a deformation-based proxy for injury.

- Cases in which the inanimate object (a) is of roughly the same mass as the relevant part of the human body, or (b) is of roughly the same stiffness, or (c) is moving so fast that human acceleration is relevant even though the human is larger, or human deformation is relevant even though the human is stiffer. For simplicity, these cases were omitted from consideration, as mentioned in section 3.2.

There are some lines of approach available for cases of the third type.

- When both masses need to be taken into account, the law of conservation of momentum can be used to work out their respective changes of velocity. There is a little more on this in section 4.2.3.
- When both stiffnesses need to be taken into account, it may be possible to work out a single stiffness that is equivalent. There is a little more on this in section 5.2.2, where a car's bonnet and a pedestrian's head are treated as being two springs (not necessarily linear) in series. There is an example in section 18.13.

Keeping a clear head about what is accelerating, what is deforming, and what is the source of injury or damage is not always easy, especially when both the human and the inanimate object may be accelerating or deforming. I hope that I have not blundered too badly in this book. My main wish, as I say in sections 1.1 and 1.3, is to improve data description and data analysis.

## *3.8 Discussion*

Cushions protecting against impact should be tested, and testing must use some measurement in place of injury. It is important to choose a measurement that is as appropriate as possible for the type of injury envisaged.

The importance of biofidelity is widely appreciated, but the work of many researchers is largely restricted to one discipline or one source of injury or one part of the body. Biofidelity is a factor limiting generalisation beyond a specific setting. The results in Hutchinson (2013) referred to a small rigid human. In order to throw light on their possible wider relevance, this chapter has clarified the importance of realism in testing. If a dependent variable based on deformation (such as maximum deformation, or maximum Viscous Criterion) is in use, the method for measuring deformation (e.g., a physical dummy, or a mathematical model) needs to be biofidelic in regard to deformation. If a dependent variable based on acceleration (such as maximum acceleration or HIC) is in use, the method for measuring acceleration needs to be biofidelic in regard to acceleration.

The various proxies for injury are affected by the conditions of the impact, such as its speed, the mass of the object that accelerates (e.g., a pedestrian's head), and the stiffness of the deforming surface (e.g., a car's bonnet). This will be shown in chapter 4.

The effects of the conditions of impact are different for the different proxies for injury. The effects may even be in opposite directions.

- HIC and maximum acceleration are quite similar concepts. Both are based on translational movement, not rotational; both are based on acceleration, not force or deformation or something else. Even so, an example will be given in section 6.5.4 of a change in conditions giving a reduction in maximum acceleration and an increase in HIC.
- Despite the example in section 6.5.4, it will usually be the case that a change in conditions changes maximum acceleration and HIC in the same direction. It is much more likely that a concept based on force or on deformation will behave differently from one based on acceleration.
- The deformation just referred to is of the human, and is a possible proxy for injury. Sometimes deformation is of the inanimate object, and is not a proxy for injury. For example, high stiffness of a car bonnet is likely to lead to low deformation of the bonnet and high acceleration of a pedestrian headform. See sections 4.5.1 and 7.5.

In the context of damage occurring in two idealised situations to manufactured items, Suhir (1997) compared equations for maximum dynamic stress and for $A_{max}$. He finds these quantities are proportional, with the important proviso that the constant of proportionality depends on mass and geometric properties. Geometric properties are often outside the scope of discussion, but mass is often of interest. The paper by Suhir is also noteworthy for discussing the possibility that dependent variables that might be considered as approximately equivalent in many situations may in fact be differently affected by certain independent variables.

It is highly desirable to know which proxy for injury is most suitable. See, for example, King (2004). My impression is that there is considerable uncertainty among experts as to what concept comes closest to representing what causes injury.

- Martin et al. (1994) prefer acceleration, and I think that preference has been common for some decades. However, part of the reason may be the practicability of measuring acceleration with instrumented headforms and similar devices.
- For injury to soft tissues, including the brain, many people think that deformation, and perhaps rate of deformation, are important.
- For fracture of bone, including the skull, many people think that maximum force is important.

Newman (1987) was quite critical of HIC.

- "HIC could not ever correlate well to skull fracture for skull fracture does not, in general, depend upon acceleration. Fracture of bone

usually comes about from the imposition of stresses induced by bending (or twisting) moments caused by the local application of force."
- "Similarly, HIC could not ever correlate well to brain injury for brain injury is not, in general, caused by translational acceleration. Brain injury is generally considered to be associated with distortion of the brain within the cranial cavity; a phenomenon which appears to be largely associated with rotational movement of the head."

As I understand these criticisms, they apply to any acceleration-based quantity, as well as to HIC. They apply to $A_{max}$, for example.

When considering the difficult question of what concept comes closest to representing what causes injury, we should also have in mind that even measurement of roughly the same thing by two roughly similar methods can give rather different results. I am thinking of a comparison of motorcycle helmet test results, according to two tests referred to as FMVSS No. 218 and ECE R22. Sixteen helmets were compared by Rigby et al. (2011, Figure 12).
- Concerning maximum acceleration, there was agreement in the sense that the average of the helmets was about 200 g in both tests. However, correlation between the two values of peak acceleration was 0.11.
- Concerning HIC, there was agreement in the sense that the average of the helmets was about 1500 in both tests. However, correlation between the two values of HIC was 0.15.

(Correlations as small as these are much smaller than any correlation that can be perceived on a scatterplot.)

I fear that for many years to come, progress may be very slow on two important questions. What physical quantity most closely reflects severity of injury? What is the probability of death at various values of that physical quantity? In addition, the issue of choice of proxy for injury is only one question among several. Other examples include the probable clinical implications of a specified value of HIC or maximum acceleration, whether the clinical meaning is the same for people of different head mass, whether extrapolation to different conditions of impact is valid, the accuracy or inaccuracy in experimental results, and so on. In view of all the other uncertainties, perhaps the issue of the proxy for injury should not be allowed to hold us back. Even if we knew what really mattered as regards brain injury, it would probably not be the same for skull injury. Perhaps it is necessary to treat predictions of the effect of change in conditions of impact as quite rough approximations.

# 4. Effects of speed (and other variables) on HIC (and other variables)

## *4.1 Introduction*

### 4.1.1 Impact

When it is foreseen that something may strike or be struck by a human, a system of impact testing is often set up for that object, including testing of any padding or cushioning that it has. Examples include the interior of a vehicle, the bonnet (hood) and other parts of a vehicle exterior, a helmet lining, and the ground. The properties of these objects, such as stiffness, are very important in protecting the human from injury.

In the case of head injury, the type of impact test principally envisaged is that of an instrumented free-flight headform striking the exterior of a car (see chapter 2). The headform measures an acceleration pulse over the milliseconds of impact. The most frequently-used summaries of the acceleration pulse are maximum acceleration, the Head Injury Criterion (HIC), and maximum displacement (that is, maximum distance of deformation). Maximum acceleration and HIC are believed to reflect likely injury severity and risk of death, and so may be considered useful proxies for injury. This chapter does not claim that one class of proxies for injury, such as those based on acceleration, has been established as best able to reflect how injury is caused. The attitude is to accept formulae that (with some justification but without any very definite proof) have been used as proxies for injury.

Similarly, some objects may penetrate into a human, and a commonly-used test measurement is depth of penetration into something like gelatin that is thought to be similar to a human. Even though this book is mostly about blunt injury, there will be something about penetrating injury in chapters 15 and 16. And similarly again, damage to objects is considered in chapters 17 and 18.

Surprisingly little seems to be known about how output quantities such as maximum acceleration, HIC, and maximum displacement (or penetration) depend on inputs (conditions of the impact) such as speed, mass, and diameter. I am thinking of the mass and diameter of a pedestrian headform, and the language used here will sometimes reflect that. Free-flight headform tests are also used for vehicle interiors. For other types of test, such as the use of dummies representing the whole human in vehicle crash tests, and guided drop tests on to an anvil as in helmet testing, some modification to the results obtained may be necessary.

My impression is that quite a lot of the engineering literature on impact is not very relevant to testing with headforms. The situation analysed is typically a steel ball bearing bouncing off a steel plate. In such a case, the impact typically ends within a fraction of a millisecond, and the coefficient of restitution is close to 1. In contrast, when a headform (representing a pedestrian's head) strikes a car exterior, the impact may last for 10 milliseconds and the coefficient of restitution is about 0.25 (section 3.5.2 of Dutschke, 2013).

Notation will be as in Table 4.1. The wording refers to a headform striking a vehicle bonnet (hood). For some symbols, slightly different wording is needed for other contexts, such as a projectile moving through soft tissue (see chapters 15 and 16) or a packaged item being dropped (see chapters 17 and 18).

There are at least two contexts where relationships of output variables to input conditions are important: (a) real-world consequences of impacts in different conditions; and (b) more specifically, in impact testing, the calculation of equivalences between tests conducted with different choices of conditions.

Chapter 18 of Hutchinson (2018) is a shorter version of this chapter.

### 4.1.2 Differential equations

The starting point in section 4.2 below is to assume a specific differential equation relates the force at any instant during the impact to instantaneous displacement (deformation) and instantaneous velocity. Consequences are then derived mathematically. The simplest assumption is that of a linear spring; more complex and realistic assumptions, including nonlinearity of the spring and damping being present, are also considered. It will be shown that if the differential equation is a correct description of reality, maximum acceleration and HIC are proportional to power functions of initial velocity and mass of headform, and expressions are obtained for the exponents.

Consider a normal (perpendicular) impact of a headform with a car exterior (and assume that an angled impact may be represented by the normal component of the velocity). The force on the headform at any moment is assumed to depend on the distance travelled after first contact (i.e., the deformation of the exterior) and on velocity. The acceleration of the headform is the ratio of force to mass. Hence the differential equation is $m.x''$ = some function of $x$ and $x'$, with the initial conditions at time = 0 being $x(0) = 0$ and $x'(0) = v$. It is also understood that force becomes zero after the headform and vehicle part contact. Several possibilities for the function of $x$ and $x'$ will be considered.

The use of a differential equation is desirable because (if it is sufficiently near correct) it represents causation, and so will permit inputs such as speed and mass to be connected to outputs such as maximum acceleration and HIC.

### 4.1.3 Pulse algebra

Use of a differential equation may be contrasted with what might be called pulse algebra. This term refers to making an assumption about the shape of the acceleration pulse x"(t), and then using algebra to derive consequences from that. The consequences will be expressed in terms of the height and length of the pulse, i.e., the maximum acceleration and the time the impact lasts. Examples of shapes might include the pulse being a square wave (i.e., acceleration is constant during the period of contact, and zero otherwise), or half a cycle of a sine wave (as it is in the case of a linear spring), or a triangle (i.e., linear increase from zero followed by linear decrease to zero). Data from an impact test gives the acceleration pulse, so this is a very natural starting point. But the disadvantage is that it is necessary to assume the shape of the pulse remains the same when conditions change, and there is apparently no strong reason to accept this assumption.

Pulse algebra is useful in showing that provided pulse shape does not change (except for linear stretching in time and height), then various results follow. For example, HIC is proportional to $A_{max}^{2.5}$.T (section 4.5.1 below). Thus HIC is proportional to $A_{max}^{1.5}$ multiplied by change of velocity (deltaV), and to $A_{max}^{1.5}$.v if coefficient of restitution is constant. For some special cases of this, see Chou and Nyquist (1974).

### 4.1.4 Structure of this chapter

Section 4.2 will introduce some differential equations and will obtain some proportionality results relating outputs to inputs. Consideration of the equation of Hunt and Crossley (1975), begun in section 4.2, continues in section 4.3. When a number of important inputs are varied, the shape of the acceleration pulse remains constant, except for linear stretching or compression in time (horizontal axis) and in acceleration (vertical axis): this is established in section 4.4. The consequences of this are proportionality relationships for outputs in terms of inputs, and these are obtained in section 4.5. Section 4.6 ends the chapter with comments on some topics that arise (and chapter 5 will deal with more of them).

Table 4.1. Notation used in chapters 4 and 5. As mentioned in section 1.9, it may be the case that a symbol is used to mean one thing in one chapter of the book and another in another chapter.

| | |
|---|---|
| $A_{max}$ | maximum acceleration during the impact |
| b | damping constant |
| c | a distance |
| d | headform diameter (in principle, d might represent any other relevant factor) |
| deltaV | change of velocity |
| $F_{max}$ | maximum force during the impact |
| HIC | the Head Injury Criterion (see section 4.2.1) |
| k | spring stiffness |
| $k_2$ | damping constant |
| m | mass of headform |
| n, p, q | exponents |
| R | coefficient of restitution |
| s | an exponent |
| S | maximum displacement (depth of penetration in chapters 15 and 16) |
| t | time since the start of impact |
| T | total contact time of the impact |
| $T_0$ | the time at which speed is zero and displacement is at its maximum |
| u, w | exponents in a general injury response function (section 4.6.2) |
| v | initial velocity at impact (assumed to be normal to the surface) |
| $v_c$ | a velocity |
| x(t) | displacement of the headform into the vehicle's bonnet |
| x'(t) | first differential, velocity |
| x''(t) | second differential, acceleration |
| y(t) | a function of time, that is considered (section 4.4) as a solution of the differential equation in x |
| y'(t) | first differential |
| y''(t) | second differential |

## *4.2 Differential equations, and some straightforward results*

### 4.2.1 Notation

Notation will be as in Table 4.1.

The Head Injury Criterion HIC is $[av(a)]^{2.5}.(t_2 - t_1)$, where av(a) is average acceleration over a period from $t_1$ to $t_2$, with $t_1$ and $t_2$ chosen so that the resulting HIC is maximized, and average acceleration is velocity change in the relevant period divided by ($t_2$ - $t_1$). (It is sometimes required that ($t_2$ - $t_1$) does not exceed a prespecified length of time, e.g., 15 msec. This detail will be ignored.)

### 4.2.2 Outputs from a differential equation

It is desired that a differential equation will allow implications concerning maximum acceleration and HIC to be obtained, and perhaps also concerning maximum displacement S and the coefficient of restitution R.

S and R are, like $A_{max}$ and HIC, outputs from an impact test, not controlled conditions like headform mass and initial velocity are. Nevertheless, they may be regarded as simpler than $A_{max}$ and HIC, and it may be desired to find equations for $A_{max}$ and HIC in terms of S and R: a greater S probably reflects a less stiff structure, and therefore less severe injury, and a greater R implies a greater change of velocity, and therefore more severe injury.

However, it is necessary to be careful when considering relationships between outputs (dependent variables).

- The relationships depend on what is causing them: that is, on what variable is it whose variation is causing the co-variation of the outputs. For this, see especially sections 6.5 and 6.6.
- It seems that relationships between two outputs may be subject to much greater random error than the relationships of either output to an independent variable such as v. For the relationship between $A_{max}$ and HIC, see Appendix 5.

S is of particular interest because there may be only limited space for deformation before something much stiffer is encountered. Furthermore, in other contexts (see chapter 3), displacement (also termed deformation or deflection) may refer to the human rather than to the vehicle bonnet, and may be a direct indicator of likely injury. (For strain of the skull, see section 5.3.) The same might be true of the product of displacement and rate of change of displacement, x.x' (see chapter 13). Measures of these types are particularly relevant to chest injury (see, for example, Digges, 1998).

There might be an attempt to split S into components, such as local and global. (These terms might respectively refer to within and outside the contact area.) And the forces acting might be thought of as local or global. I do not think this distinction is at all common. See section 17.12, however.

Differential equations are often used to obtain explicit equations for displacement x as a function of time, and similarly the acceleration pulse x'' as a function of time. This is done below in only a few special cases (see sections 4.2.3 and 4.6.3); otherwise, useful results are obtained without an explicit expression for x''(t).

### 4.2.3 Linear spring

In the case of the linear spring, the term in velocity x' is absent, and the differential equation is $x'' + (k/m).x = 0$. That is, force is proportional to displacement. To solve this is elementary: the acceleration pulse is half a cycle of a sine wave.

This differential equation is not equivalent to the acceleration pulse being half a cycle of a sine wave, however. In particular, the differential equation implies that the coefficient of restitution is 1, whereas algebraic results can be obtained from this shape and a range of velocity change (and thus a range of coefficient of restitution). As already noted, the coefficient of restitution is about 0.25 for headforms hitting bonnets: close to 1 is unrealistic.

Searson et al. (2009) found that HIC is proportional to $m^{-0.75}.v^{2.5}$. Results for the half sine pulse with other coefficients of restitution are given in section 5.2 of Dutschke (2013).

As noted in section 3.7, when the masses of both colliding objects need to be taken into account, the law of conservation of momentum can be used to work out their respective changes of velocity. The proportionality relationship of Searson et al. (2009) refers to an object (e.g., a head) of mass m and speed v striking a stationary object that is much more massive (e.g., a car bonnet). It is sometimes the case that with two modifications, such a relationship can be transformed into one that refers to an object of mass $m_2$ and speed v striking a stationary object of mass $m_1$. The first modification is that v is replaced by velocity change, which is proportional to $(m_1/(m_1 + m_2)).v$. The second is that m is replaced by the so-called reduced mass, $(m_1/(m_1 + m_2)).m_2$. This would suggest that the proportionality relationship given by Searson et al. (2009) might become $(m_1/(m_1 + m_2))^{-0.75}.m_2^{-0.75}.(m_1/(m_1 + m_2))^{2.5}.v^{2.5}$, which is $(m_1/(m_1 + m_2))^{1.75}.m_2^{-0.75}.v^{2.5}$. For such an expression, see equation (8) of Gao and Wampler (2009).

The relationship between compressive stress and strain is commonly shown as proportionality at low stress and strain, a plateau where stress only gradually increases, and a sharp increase of stress at high strain. This may be approximated by force being a piecewise linear function of displacement having three regimes, the second being the least steep and the third being the steepest. That model was used by Deb and Ali (2004) in simulating headform impacts with car interiors; results computed included acceleration pulses and values of HIC.

### 4.2.4 Nonlinearity of the spring

A simple way of making the spring nonlinear is to assume force is a power function of displacement, $x'' + (k/m).x^n = 0$. This permits the spring to become either more or less stiff as displacement increases (if $n > 1$, or $n < 1$, respectively). This generalisation of linearity might be thought so simple that it needs no justification. Or a justification might be found in the modelling of plastic behaviour. When stretching a solid by a force, it is found that many materials are linearly elastic at first, and then become plastic. The relationship of stress to plastic strain is often regarded as a power function; an alternative is that stress equals yield stress plus an amount that is proportional to a power of plastic strain. The hardness of materials largely reflects plastic properties, and power functions are often found in that context also.

In terms of the description in the final paragraph of section 4.2.3, some value of n less than 1 might adequately approximate both proportionality at low stress and strain and a plateau, and some value of n greater than 1 might adequately approximate both a plateau and a sharp increase.

> The unattractive feature of a power curve is the qualitatively different dependence on x when x is close to 0 for the three cases $n = 1$, $n > 1$, and $n < 1$. That is, a power curve does not have the desirable property that when x is close to 0, it is only slightly different from the case $n = 1$.

Some examples of results for power-function resistance are as follows.

- Martin (1990) and Martin et al. (1994) were chiefly concerned with athletes' impacts with playing surfaces, and Ohue and Miyoshi (2014) with playground surfaces. The opinion of Martin (1990, p. 80) was that for playing surfaces impacted by an adult, a value of exponent n of about 3 was typical.
- In some circumstances, the case $n = 1.5$ is important, arising from linearity between stress and strain along with the geometry of a sphere, and is termed Hertzian impact (see article 142 of Timoshenko and Goodier, 1970).
- Jeong et al. (2011) were concerned with massive objects striking railway tank cars (wagons). They referred to earlier work in which $n = 1.5$ was assumed, but they regarded $n = 0.5$ as more realistic. This was based in part on the theoretical work of Wierzbicki and Suh (1988).
- Consider a ship colliding head on with a rigid wall. Zhang et al. (2004) considered that for deformations limited to near the bow, the force would be proportional to $x^{0.5}$. (The reason was that, in plan view, a ship's bow is approximately a parabola, and so the breadth is approximately proportional to the square root of the distance from the extreme bow.) They found that distance crushed (and

distance damaged) would be proportional to $(m.v^2)^{2/3}$. This is consistent with what will be found in section 4.5 below.

- The case n = 0 was considered by Neilson (1969), in the contexts of padding that might be struck by a car occupant and of crush of a car's front. This may have been partly for the tractability of the algebra. But Neilson gives some attention also to resistance that decreases with crush, so I think it more likely that Neilson considered n = 0 of genuine interest. For n = 0, maximum force and maximum acceleration are minimised, for a given amount of energy absorption.
- Mindlin (1945) considered a packaged item in a container as one mass connected to another by a spring, and analyzed the motion of the item for various types of spring. Mindlin considered several possible equations for force-displacement relationships. See chapter 17 for further discussion.

For power-function resistance, some of the desired results may be obtained by the elementary method of equating the kinetic energy of impact to the energy absorbed. (For another example of this method, see section 17.4.1.) Let S be maximum displacement (maximum deformation).

- Energy is force integrated over distance, that is, $k.x^n$ integrated from 0 to S, which equals $k.S^{n+1}.(1/(n+1))$. This must equal kinetic energy of impact, $\frac{1}{2}.m.v^2$. Therefore S is proportional to $(m/k)^{1/(n+1)}.v^{2/(n+1)}$. Tang et al. (2008) had this result.
- Force increases with displacement, and maximum force occurs when displacement is at its maximum. Thus maximum force is $k.S^n$, and is proportional to $k.(m/k)^{n/(n+1)}.v^{2n/(n+1)}$. Tang et al. (2008) and Jeong et al. (2011) had this result. So do Perera et al. (2016) and Sun et al. (2015), though they consider that k and n might vary with impact speed v.
- Maximum acceleration is maximum force divided by mass, and is proportional to $(m/k)^{-1/(n+1)}.v^{2n/(n+1)}$.

Thus kinetic energy is proportional to maximum force $F_{max}$ to the power (n + 1)/n.

For n = 1 (linear spring), this is $F_{max}^2$, and for n = 1.5 (Hertzian impact), it is $F_{max}^{5/3}$. If both these forces are acting --- for example, there might be both large-scale deformation having n = 1 and deformation localised at the contact point for which n = 1.5 --- kinetic energy will be proportional to a weighted average of $F_{max}^2$ and $F_{max}^{5/3}$ (Jang et al., 2002).

A result for HIC cannot be obtained so easily (as far as I know).

As discussed in sections 2.3 and 2.4, bottoming out refers to contact between the underside of the bonnet (or other surface structure) and some much stiffer structure beneath (such as the engine). This is potentially very serious: there may be a great increase in HIC, maximum acceleration, and likely injury. If bottoming out is thought to be possible, then force being proportional to a power function of displacement, with the exponent n being bigger than 1, will be more realistic than linear resistance. However, an even steeper increase of force might be a further improvement, as with a function that tends to infinity for some finite displacement. The function $\tan(x^*.(\pi/2))$ (where $x^*$ is the ratio of x to the maximum available displacement), referred to as tangent elasticity, is one possibility. Section 17.4 will consider the function $1/(1 - x^*)$

The proportionality results to be given below will no longer be valid when there is bottoming out; if a good model of bottoming out is available, relationships might be obtainable via numerical simulation instead.

### 4.2.5 A multiplicative damping term (Hunt and Crossley)

A simple way of generalising the linear spring to include a velocity-sensitive term (damping) is $x'' + (k_2/m).x' + (k/m).x = 0$. A friction term (constant force in the opposite direction to velocity) may also be included. In the context of packaging (see chapter 17), that has been termed the friction-viscous damping (FVD) model, and some results for maximum acceleration obtained by Piao et al. (2017).

However, it is sometimes argued that this is unrealistic, and that the damping term should be 0 for both $x = 0$ and $x' = 0$ (Hunt and Crossley, 1975). That suggests a product term in powers of x and x'. A simple example incorporating a nonlinear spring is

$$x'' + (k/m).x^n.(1 + (b/v).x') = 0$$

(Here, $n > 0$ if the damping term is required to be 0 for $x = 0$, but otherwise $n = 0$ might be included.) It is understood that b, k, and n remain constant, not only as time t passes, but also if v and m change. A factor $v^{-1}$ is included in the damping term because then constancy of b implies constancy of coefficient of restitution when v changes, and that is thought to be approximately true empirically: see pp. 212-213 of Gonthier et al. (2004). A damping term proportional to $x^p.(x')^q$ was proposed by Hunt and Crossley, and they gave particular attention to the case $p = n$, $q = 1$, as in the equation above. Han (2007) suggested the equation be used to model front-to-rear vehicle impacts, and Anderson et al. (2009) suggested it be used to model pedestrian-vehicle contact. The $v^{-1}$ factor may date from Herbert and McWhannell (1977). Models of this type are reviewed by Flores and Lankarani (2016) and Banerjee et al. (2017). In many of the models, x' appears in the form of the ratio $x'/v$. Jacobs and Waldron (2015) give some results for the model in which $v^{-1}$ does not appear.

It will be convenient to refer to this as the Hunt and Crossley model or equation. I am not sure, however, whether that is quite appropriate: the algebraic expression is similar, but not the context of application. Hunt and Crossley had in mind impacts such as solid steel with solid steel, and coefficients of restitution above 0.84 and perhaps much closer to 1 than that. In contrast, as mentioned in section 4.1.1, the coefficient of restitution is approximately 0.25 when a headform hits a car bonnet.

For the above differential equation, m and v change the height and length of the acceleration pulse but do not otherwise change its shape, and some proportionality results concerning $A_{max}$, HIC, and S are available (Hutchinson, 2013).

- $A_{max}$ is proportional to $(m/k)^{-1/(n+1)}.v^{2n/(n+1)}$.
- HIC is proportional to $(m/k)^{-1.5/(n+1)}.v^{(4n+1)/(n+1)}$.
- Maximum displacement S is proportional to $(m/k)^{1/(n+1)}.v^{2/(n+1)}$.

These results refer to any particular differential equation of the given form, i.e., to any location of impact where this equation is valid. For the case n = 0, see the final paragraph of section 4.6.3.

Some results of other types --- e.g., illustrations of the acceleration pulse for various values of the spring exponent and the damping constant, and of the dependence of the coefficient of restitution on the damping constant --- have been obtained by simulation (Searson et al., 2012b).

## *4.3 Further consideration of the Hunt and Crossley equation*

### 4.3.1 Generalisation

From the Hunt and Crossley equation, Hutchinson (2013) obtained proportionality results as described in section 4.2.5. Four generalisations or extensions will be given in this section.

- In the damping term, the exponent of x will be permitted to be something other than n.
- The exponent of x' will be permitted to be something other than 1.
- It will be shown that if some other variable is introduced into the equation in a specific way, then maximum acceleration, HIC, and maximum displacement will be power functions of it.
- The class of proxies for injury (or injury response functions, or injury metrics) will be expanded beyond maximum acceleration and HIC. The exponent 2.5 used in calculating HIC originated decades ago, based on limited data. Suppose, instead, the relevant injury response function is based on $[av(a)]^u.(t_2-t_1)^w$. It will be shown that this injury response function also has power-function dependence on initial velocity v and headform mass m.

The generalised form is as follows.

$$m.x'' + b.x^p.(x')^q.v^{(2(n-p)/(n+1))-q}.m^{(n-p)/(n+1)}.d^{s(p+1)/(n+1)} + k.x^n.d^s = 0$$

In the second term of the equation, the exponent of x is p, and the exponent of x' is q, as in the general form of the proposal in Hunt and Crossley (1975). In case the other exponents in the second term are not clear, they are now listed:

the exponent of v is (2.(n - p) / (n + 1)) - q,
the exponent of m is (n - p) / (n + 1),
the exponent of d is s.(p + 1) / (n + 1).

An additional variable d has been introduced: this could represent headform diameter, but it might be anything. Unlike the other exponents, s is not necessarily positive.

This equation is perhaps unattractive itself, as there seems to be no reason why the damping term should depend on m. This will be irrelevant if m is constant in a particular context. And the equation subsumes a number of more plausible equations as special cases.

In the second term, the exponents of v, m, and d have been chosen in such a way that the shape of the acceleration pulse remains constant, except for horizontal and vertical linear stretching, when v, m, and d change (see section 4.4). And if that is true, then (as in Hutchinson, 2013) maximum acceleration, HIC, and maximum displacement are all power functions of initial velocity v, headform mass m, and factor d, the exponents depending on the spring exponent n. (In the case of factor d, its exponent also depends on s.)

### 4.3.2 Special cases (1)

In the absence of damping (b = 0) the equation is as follows.

$$m.x'' + k.x^{n}.d^{s} = 0$$

For Hertzian impact, n = 1.5 and s = 0.5, with d being headform diameter (article 142 of Timoshenko and Goodier, 1970).

### 4.3.3 Special cases (2)

Setting p = n leads to

$$m.x'' + b.x^{n}.(x')^{q}.v^{-q}.d^{s} + k.x^{n}.d^{s} = 0$$

If q = 1 and s = 0, the equation in section 4.2.5 is obtained. This has been used quite widely.

### 4.3.4 Special cases (3)

If m and d do not vary, the relevant factors can be absorbed in the constants, and

$$x'' + b.x^{p}.(x')^{q}.v^{(2(n-p)/(n+1))-q} + k.x^{n} = 0$$

If p = 0,

$$x'' + b.(x')^{q}.v^{(2n/(n+1))-q} + k.x^{n} = 0$$

The damping term is now no longer 0 for x = 0. If q = 2.n/(n + 1),

$$x'' + b.(x')^{2n/(n+1)} + k.x^{n} = 0$$

An exponent n = 1.5 might be thought realistic, and so might be an exponent q = 1.2 for the x' term.

### 4.3.5 Comments

Again suppose that m and d do not vary. Because of its importance for injury, what is of chief interest is x'' when x'' is close to its maximum. Realistically, this occurs before maximum displacement occurs, that is, when displacement is increasing and speed is decreasing. It seems likely that some example of the differential equation will be a good approximation, as $x^{p}.(x')^{q}$ is the product of an increasing term and a decreasing term, and p and q can be adjusted so that this product has a maximum. If it is thought that the damping term should not depend on v, then q must equal 2.(n - p)/(n + 1). For example, this is so if n = 1.5, p = 0.25, and q = 1. With four constants that can be adjusted, it seems very likely that some choice of them will mean that $b.x^{p}.(x')^{2(n-p)/(n+1)} + k.x^{n}$ is a good approximation to x'' in the relevant range.

The argument in the previous paragraph is not relevant to maximum displacement S, which is not determined largely by the behavior of the equation when x'' is close to its maximum.

The case of m, d, and v not varying will be of interest when various impact locations are tested to the same test protocol. For this, see section 6.5.5.

The set of exponents n = 1.5, p = 0.5, q = 1, s = 0 has had some attention in the literature. However, the equation under discussion requires $v^{-0.2}.m^{0.4}$ as a multiplier of the damping term, and consideration may not have been given to this. Similarly, n = 1, p = 0, s = 0, and q = 1 or 0 (respectively, Kelvin-Voight damping and dry friction) might be considered to be special cases, except that $m^{0.5}$ and $v.m^{0.5}$ do not usually appear as multipliers of the damping term. For some analysis of the effects of damping and friction, see sections 2.5 and 2.7 of Mindlin (1945).

In addition to concentrating on the special cases, it might be sufficient to only consider the part of the deceleration pulse preceding maximum displacement, and restrict the differential equation to that time; the coefficient of restitution can then be treated as an independent variable.

## *4.4 Constancy of the shape of the acceleration pulse*

It will be shown that for the differential equation in section 4.3.1, the shape of the acceleration pulse is constant, except for horizontal and vertical linear stretching or compression.

If v = 1, m = 1, and d = 1, the equation referred to is as follows.

$x'' + b.x^p.(x')^q + k.x^n = 0$

If y(t) is a solution to this, then

$d^{-s/(n+1)}.m^{1/(n+1)}.v^{2/(n+1)}.y(d^{s/(n+1)}.m^{-1/(n+1)}.v^{(n-1)/(n+1)}.t)$

is a solution to the equation in section 4.3.1. As this takes the form of a constant multiplied by y (i.e., a vertical linear stretching or compression), with the argument of y being a constant multiplied by t (i.e., a horizontal linear stretching or compression), the acceleration pulse will also change only by vertical and horizontal linear stretching or compression.

The reasoning, similar to that in Hutchinson (2013), is as follows.

- x is assumed to be $d^{-s/(n+1)}.m^{1/(n+1)}.v^{2/(n+1)}.y(d^{s/(n+1)}.m^{-1/(n+1)}.v^{(n-1)/(n+1)}.t)$.
- Differentiate once: $x' = v.y'$.
- Differentiate again: $x'' = d^{s/(n+1)}.m^{-1/(n+1)}.v^{2n/(n+1)}.y''$.
- Introduce the above expressions for x, x', and x'' into the left hand side of the differential equation.
- The multiplier of y'' is $d^{s/(n+1)}.m.m^{-1/(n+1)}.v^{2n/(n+1)}$, which is $d^{s/(n+1)}.m^{n/(n+1)}.v^{2n/(n+1)}$.
- The multiplier of $b.y^p.(y')^q$ is $d^{-sp/(n+1)}.m^{p/(n+1)}.v^{2p/(n+1)} \times v^q \times v^{(2(n-p)/(n+1))-q}.m^{(n-p)/(n+1)}.d^{s(p+1)/(n+1)}$, which is $d^{s/(n+1)}.m^{n/(n+1)}.v^{2n/(n+1)}$.
- The multiplier of $k.y^n$ is $d^{-sn/(n+1)}.m^{n/(n+1)}.v^{2n/(n+1)}.d^s$, which is $d^{s/(n+1)}.m^{n/(n+1)}.v^{2n/(n+1)}$.
- Thus $d^{s/(n+1)}.m^{n/(n+1)}.v^{2n/(n+1)}$ is a common factor. It multiplies the rest of the expression, which is $y'' + b.y^p.(y')^q + k.y^n$. But this is known to be 0. Consequently the left hand side of the differential equation in section 4.3.1 has been shown to be 0.

## *4.5 Consequences of the constancy of pulse shape*

### 4.5.1 Proportionality relationships

From the above expressions for x and x", the following proportionality results for maximum displacement S and maximum acceleration $A_{max}$ are immediately apparent.

$S \propto d^{-s/(n+1)}.m^{1/(n+1)}.v^{2/(n+1)}$

$A_{max} \propto d^{s/(n+1)}.m^{-1/(n+1)}.v^{2n/(n+1)}$

In respect of m and v, these are the same results as in Hutchinson (2013).

Given that pulse shape is constant except for horizontal and vertical linear stretching, HIC will be proportional to $A_{max}^{2.5}.T$, where $A_{max}$ is maximum acceleration and T is the time for which the pulse lasts (Hutchinson, 2013). Thus HIC is proportional to

$[d^{s/(n+1)}.m^{-1/(n+1)}.v^{2n/(n+1)}]^{2.5}.[d^{s/(n+1)}.m^{-1/(n+1)}.v^{(n-1)/(n+1)}]^{-1}$

and so

$HIC \propto d^{1.5s/(n+1)}.m^{-1.5/(n+1)}.v^{(4n+1)/(n+1)}$

Table 4.2. Exponents of (m/k) and v, for maximum acceleration, pulse duration, HIC, and maximum displacement.

| | Exponent of (m/k) | Exponent of v |
|---|---|---|
| Maximum acceleration $A_{max}$ | -1/(n + 1) | 2.n/(n + 1) |
| Duration T | 1/(n + 1) | -(n - 1)/(n + 1) |
| HIC | -1.5/(n + 1) | (4.n + 1)/(n + 1) |
| Maximum displacement S | 1/(n + 1) | 2/(n + 1) |

From the way that m and k appear in the differential equation, m should be interpreted as m/k in these proportionality relationships, as in Hutchinson (2013).

For convenience, the exponents of m/k and v are listed in Table 4.2.

Maximum displacement S may be considered simpler than maximum acceleration and HIC, and it may be desired to have results for maximum acceleration and HIC in terms of maximum displacement.

- On the assumption that the shape of the acceleration pulse is quadratic, Mizuno and Kajzer (2000) algebraically demonstrated that HIC $\propto S^{-1.5}.v^4$.
- On the assumption of an asymmetric haversine pulse (as they call it), Zhou et al. (1998) algebraically demonstrated that HIC $\propto S^{-1.5}.v^4$.
- Figure 9 of NHTSA (1993), said to be the result of theoretical analysis, shows HIC versus S for three speeds. I have attempted to calculate what relationship underlies that Figure, and it is fairly clear that it is HIC $\propto S^{-1.5}.v^4$.

The relationships given earlier show that the proportionality relationships found by Mizuno and Kajzer and Zhou et al. hold in more general conditions.

- Maximum acceleration is proportional to $S^{-1}.v^2$.
- HIC is proportional to $S^{-1.5}.v^4$.

These relationships will hold if m or k change. They will not apply if b or n change. Referring to different impact surfaces, it might be adequate to characterise many impact surfaces (or at least those of a particular type) by k alone, with b and n being constant.

### 4.5.2 Energy

Maximum force may also be of interest. It will be proportional to $m.A_{max}$, that is, to $m.m^{-1/(n+1)}.v^{2n/(n+1)} = m^{n/(n+1)}.v^{2n/(n+1)}$. The effects of m and v on S and maximum force are captured by the effect of kinetic energy, $½.m.v^2$. That is, S is proportional to energy$^{1/(n+1)}$, and maximum force is proportional to energy$^{n/(n+1)}$.

As regards m and v (that is, assuming that k does not change), the following may be noted. Firstly, if n is 1, S and maximum force are proportional. Secondly, even if n is not 1, S and maximum force are functions of each other. Consequently, it may sometimes be the case that a threshold or limiting or ultimate value of S could be replaced by a threshold or limiting or ultimate value of maximum force, or vice versa.

Experiments on animals may employ a direct measure of injury. An example will be mentioned in section 12.4. Another example is study of the effects of contusion of the spinal cord of rats. Ji et al. (2014) were concerned with the question of how should the impact be quantitatively summarised. They varied m (relative magnitudes 3, 2, 1) and v (relative magnitudes 2, 3, 6). For the three combinations of m and v employed (three groups of rats), the product m.v was constant, and the three values of $m.v^2$ were of relative magnitudes 2, 3, 6 (that is, 12, 18, 36). Ji et al. found that the effect of the spinal contusion (as shown by locomotor function and subsequently by histopathological assessment) was greatest for the third group of rats, the other two groups being similar.

- As the effects in the three groups were not all similar, Ji et al. concluded that momentum m.v is inappropriate as a method of quantification.
- It could be that energy, perhaps acting via maximum deformation or maximum force, is appropriate. Impact energy being relatively 2, 3, 6, the third group differed appreciably from the other two. The fact that results were similar for the two groups of rats for which $m.v^2$ differed by relatively little would then be ascribed to chance.

### 4.5.3 Discussion

The proportionality results also apply to the undamped case (section 4.2.4), of course. Consequently, if damping is thought to be negligible at the highest relevant speeds, static (non-destructive) testing might be sufficient. A single impact test could be used to fix one point on the relationship between a dependent variable (such as HIC) and independent variables (m, v, d), with the dependent variable for other values being inferred on the basis of the spring exponent n measured in a static test.

As the spring exponent n is positive, the exponents expressing outputs in terms of inputs are predicted to lie within certain ranges. For example, (4.n + 1)/(n + 1) (referring to HIC in terms of v) must be between 1 and 4. If a prediction is found empirically to be untrue, something is wrong: for example, the Hunt and Crossley equation might not apply. An obvious possible reason is the occurrence of bottoming out. Without some sort of theory, there would be no baseline like this against which results could be judged.

It is not clear whether n should be expected to be less than 1 (the surface may have been designed to be a spring that gets less stiff with increasing displacement); or to be greater than 1 (displacement may be approaching the point of bottoming out).

Could the reasoning in sections 4.4 and 4.5.1 be wrong? Yes, that is possible. I am not a mathematician, and I might have made mistakes. I think it is not likely to be very wrong, as some results could be obtained by elementary means in the case of a spring without damping (section 4.2.4). And if there is an error, it is likely that a useful theory could be constructed from the assumption of a linear spring together with some special phenomenon at initial contact (see section 5.4).

Could the reasoning be well known? Yes, that is possible, and even quite likely, as the results are simple. I do not think it is well known to specialists in vehicle safety.

## *4.6 Comments on this chapter*

### 4.6.1 Effect of headform mass

Mass of impactor has two contrasting effects. If there is no sudden bottoming out, greater mass means lower maximum acceleration. However, greater mass also means higher deformation, and thus a greater likelihood of sudden bottoming out.

Leaving aside consideration of bottoming out, headform mass m has a negative effect on maximum acceleration and HIC: higher mass means lower acceleration. This might be thought surprising, but is a consequence of these injury response functions being based on acceleration rather than force. Maximum displacement is positively related to m, as would be expected as increased m means more kinetic energy. As deceleration takes place over a longer distance, accelerations are smaller. In contrast, headform mass has a positive effect on maximum force.

In general terms, this issue is well-known, though there does not seem to be a consensus about its resolution. In a detailed modelling context (using the finite element method), Kleiven and von Holst (2002) found a negative effect of mass on HIC and a positive effect on measures of intracranial stress. Ruan and Prasad (2006) found that for a given acceleration pulse, a head having twice the mass of another received twice the force (the differences they found for stress probably in part reflected differences in skull thickness). Mertz (1993) suggested that depending on head mass, rather different values of HIC should be considered as being equivalent as regards injury. The figures at p. 81 of Mertz (1993) imply that it is more appropriate to treat the product of HIC and $m^{0.25}$ as a constant criterion than either HIC or the product of HIC and m. For discussion of HIC and other simple measures as compared with complex measures calculated from finite element models of the head and brain and based on strains and stresses, see Sanchez-Molina et al. (2012).

Another layer of difficulty arises because the clinical effect of a certain level of HIC or $A_{max}$ may differ according to head mass. Head mass is correlated with body mass, and a small head mass is likely to be that of a child or perhaps an elderly person. The clinical effect of a given level of HIC or $A_{max}$ is not likely to be the same for them as for a young adult.

### 4.6.2 Generalisation of the proxy for injury severity

HIC is based on $[av(a)]^{2.5}.(t_2 - t_1)$. Consider, instead, a proxy for injury severity, or injury response function, based on $[av(a)]^u.(t_2 - t_1)^w$. Then, given that pulse shape is constant except for horizontal and vertical linear stretching, this new injury response function will be proportional to $A^u.T^w$. Thus it is proportional to

$$[d^{s/(n+1)}.m^{-1/(n+1)}.v^{2n/(n+1)}]^u.[d^{s/(n+1)}.m^{-1/(n+1)}.v^{(n-1)/(n+1)}]^{-w},$$

which is

$$d^{(u-w)s/(n+1)}.m^{(w-u)/(n+1)}.v^{((2u-w)n+w)/(n+1)}.$$

Setting u = 2.5, w = 1, returns to the expression applicable to HIC. Setting u = 1, w = 0, returns to the expression applicable to maximum acceleration. And if u = 1, w = 2, the expression applicable to maximum displacement is obtained.

Interest lies chiefly in the clinical effects corresponding to particular values of an injury response function, and so it might be said that injury response functions should be considered to be ordinal variables rather than fully quantitative, as mentioned at the beginning of section 3.3. Thus the ratio u/w will be meaningful, but not the separate values of u and w: the quantity $A^{2u}.T^{2w}$, for example, puts a set of acceleration pulses into exactly the same order as $A^u.T^w$ does. Consequently, it is likely that for most purposes w can be taken as 1, and the injury response function is then proportional to

$$d^{(u-1)s/(n+1)}.m^{(1-u)/(n+1)}.v^{((2u-1)n+1)/(n+1)}.$$

According to Nakano et al. (2010), the choice of u = 2.5 is a way of selecting the main peak of acceleration from the noise; Nakano et al. (2011) give some consideration to u being 2, 2.5, or 3, and envisage the calculations being applied to accelerations measured at the chest or pelvis as well as at the head.

### 4.6.3 Two cases for which the acceleration pulse may be derived

The proportionality results in section 4.5.1 have been obtained in the absence of equations for the acceleration pulse as a function of time, x''(t).

A special case for which there is an elementary expression for x'' is as follows. If the undamped spring is omitted, leaving only damping, then x'' + (k.b/(m.v)).x.x' = 0. (This is obtained by setting n = 1, and letting k tend to zero and b to infinity while keeping the product k.b constant.) The coefficient of restitution is 0. The pulse is given below.

Displacement x(t) is $v.(2.m/(k.b))^{0.5}.\tanh((k.b/(2.m))^{0.5}.t)$.

Speed x' is $v.\text{sech}^2((kb/(2.m))^{0.5}.t)$.

The acceleration pulse x'' is
$-2.(kb/(2.m))^{0.5}.v.\text{sech}^2((kb/(2.m))^{0.5}.t).\tanh((kb/(2.m))^{0.5}.t)$.

The proportionality results in section 4.5.1 apply here also.

An expression in which force is independent of displacement is x'' + (k/m).(1 + (b/v).x') = 0. Here the exponent of x is set to 0 instead of 1. The pulse is given below.

Displacement x(t) is
$-(m.v/(k.b^2)).((k.b/m).t - (b + 1).v + (b + 1).v.\exp(-(k.b/(m.v)).t))$.

Speed x' is (v/b).(-1 + (b + 1).exp(-(k.b/(m.v)).t)).

The acceleration pulse x'' is -(k.(b + 1)/m).exp(-(k.b/(m.v)).t).

This is further discussed in the context of penetrating injury (see sections 15.2 and 15.8). Proportionality results for S and $A_{max}$ will be given.

### 4.6.4 A linear spring and a velocity-dependent force

For a linear spring, conservation of energy means that $½.m.v^2. = ½.k.S^2$. Suppose, now, that energy goes into damaging the spring, and that the force responsible is some function of v. The energy equation will now be $½.m.v^2. = ½.k.S^2. + g(v).S$, for some function g. If the extra term is relatively small, maximum force $F_{max}$ will still be k.S, and the equation may be solved for $F_{max}$. As I understand it, this is the core of the proposal that Feraboli and Kedward (2004) made for an expression for $F_{max}$, and they take g(v) to be a power function.

### 4.6.5 Geometric factors

In some circumstances, it might be realistic to seek a theory that includes some geometric factors describing the impact. I am not now thinking of a headform striking a bonnet, but of impacts with some form of cushion or padding. A theory used by packaging engineers uses area and thickness of the cushion: see section 17.3.1. I suggest in section 17.4.5 how the proportionality relationship for $A_{max}$ given in section 4.5.1 might include area and thickness.

# 5. Further consideration of the effects of input variables on output variables

## *5.1 Introduction*

This chapter gives some extra results closely connected with those of chapter 4. Notation is as in Table 4.1.

Sections 5.2 and 5.3 are about deformation of the head possibly being the real cause of injury (rather than acceleration, or force, or anything else). Section 5.4 suggests some variations that might supplement equations that originate from a theory expressed as a differential equation. Section 5.5 is discussion.

When the end of this chapter is reached, quite a lot of different possibilities will have been presented. Table 5.1 is offered as a summary.

## *5.2 Possible deformation of the head*

### 5.2.1 Introduction

Some people may consider that the real source of injury is probably the maximum deformation of the head, even if this deformation is very small compared with that of the car bonnet. These people will perhaps say that acceleration of a headform is used as a proxy for injury severity because it usually correlates with deformation and is more convenient to measure.

The following quotations are from Post and Hoshizaki (2012, pp. 332, 344).

> "The mechanisms of brain trauma described have one variable in common, a certain amount of measurable deformation stresses the brain tissue past the point of recovery resulting in permanent deformation and injury" (p. 332).
>
> "Brain deformation metrics are seen by researchers as dependent variables which would be more correlative to injury, however the correct variable to use for any particular type of injury has yet to be established" (p. 344).
>
> "It could be deduced that all brain injuries are linked in some form to either translational or rotational skull/brain motions and could be quantified by measuring brain deformation" (p. 344).

Rigby et al. (2011, p. 13) refer to peak skull strain as the "most biofidelic skull fracture metric". (Admittedly, the reference to skull fracture makes this quite a limited claim.) The opinion of Katagiri et al. (2012, p. 450) was that "skull fracture should be evaluated by a stress or strain-based criterion".

Table 5.1. Overview of theoretical relationships, and where to find them in this book.

Starting point

- Limitations: only blunt injury, only from translation (not rotation), only the simplest geometry of impact
- Simplifications: one of the colliding objects is much more massive than the other, one of the colliding objects is much stiffer than the other
- Human = small, rigid (see chapter 3)
- Therefore, proxy = acceleration
- Input variables are: v, m (mass of headform), k (stiffness of a location on bonnet) (k is typically not measured)
- Main output measures are: $A_{max}$ (from headform acceleration), HIC (from headform acceleration), deformation of bonnet (from double integration of headform acceleration)
- Proportionality results: see section 4.5.

Variations in the broad type of impact

- Large deformable human: see section 3.4
- Large rigid human: see section 3.5
- Small deformable human: see section 3.6

Other proxies for injury

- Deformation (strain) of the human: see sections 5.2, 5.3
- Skull Fracture Correlate (SFC): see section 5.3
- A broad class of injury response functions: see section 4.6.2
- Viscous Criterion: see section 12.5, chapter 13

Variations of the theory

- Weak surface of the inanimate object: see section 5.4.1
- Extra energy absorption at the moment of initial contact: see sections 5.4.2, 15.6, 16.1 (especially 16.1.5), 16.2
- Putting a mass into motion at initial contact: see sections 5.4.3, 6.7.4, 18.12
- Relevance of theories for penetrating impacts: see section 15.8

Bottoming out (gradual)

- Theory and cushion curves: see section 17.4

Bottoming out (sudden)

- Various comments: see sections 1.7, 2.3, 2.4, 4.2.4, Appendix 1

On the basis of experiments with a physical head model, Zhang, Pintar, et al. (2009) considered that pressure rather than strain was probably responsible for brain injury. However, that was brain injury from blast.

Results concerning head deformation will be obtained below. Representing the whole head by a single stiffness may be very unrealistic, but some people may consider that it is less unrealistic than relying on acceleration.

Injury to one part of the head as compared with injury to another part of the head (for example, skull injury versus brain injury) may be very important medically, but is beyond the scope of broad-brush theories such as those in this book.

### 5.2.2 Proportionality results

Consider the car's bonnet and the pedestrian's head as being two springs in series.

- For the car's bonnet, force = $k_1.x_1^n$, where $x_1$ is the deformation of the bonnet.
- For the head, force = $k_2.x_2^n$, where $x_2$ is the deformation of the head. It is understood that $k_2$ is much larger than $k_1$, so there is very little deformation of the head.
- As far as I know, there are no simple results available if the exponents for bonnet and head are different.
- The two springs in series are equivalent to a power-function spring having the same exponent, force = $k.(x_1 + x_2)^n$, the constant k being $(k_1^{-1/n} + k_2^{-1/n})^{-n}$. This may be shown by elementary means (Radomirovic and Kovacic, 2015). For an example, see section 18.13.

What is now of interest is the maximum of $x_2$.

- Force = $k_2.x_2^n = k.(x_1 + x_2)^n$, and this applies when $x_2$ is at its maximum, $x_{2max}$, and $x_1 + x_2$ is at its maximum, S. Thus $x_{2max}$ is $(k/k_2)^{1/n}.S$.
- For a power-function spring, a proportionality result for S was given in section 4.5.1 and Table 4.2. Except for the terms involving the stiffnesses, the proportionality result for $x_{2max}$ will be the same: $k_2^{-1/n}.k^{1/[n(n+1)]}.m^{1/(n+1)}.v^{2/(n+1)}$.
- As S is proportional to $A_{max}^{1/n}.(m/k)^{1/n}$, deformation $x_{2max}$ is proportional to $(m/k_2)^{1/n}.A_{max}^{1/n}$. Thus when m is constant, $x_{2max}$ is a power function of $A_{max}$, whatever $k_1$ and v are.
- If $x_{2max}$ is expressed in terms of $k_1$, $k_2$, m, and v, then it can be seen that $x_{2max}$ increases with bonnet stiffness $k_1$, head mass, and impact speed. That is true of force also, not acceleration (and a similar point will be made in section 11.8.1).

For convenience, two of the relationships are shown in Table 5.2. Some people may consider the third of the above points to be a strong argument for choosing $A_{max}$ as the proxy for injury.

After some algebra, another result may be found.

- Consider m and v held constant, and so for clarity they can be omitted. The result is $x_{2max} \propto S^{-1/n}.k_2^{-1/n}$. In this expression, the effect of $k_1$ has been captured in S.
- That is, large S that occurs because $k_1$ is small implies that $x_{2max}$ is small. (Of course, large S that occurs because v or m is large will imply that $x_{2max}$ is large.)

Table 5.2. Proportionality expressions for $x_{2max}$.

| | |
|---|---|
| See section 5.2.2 | $k_2^{-1/n}.k^{1/[n(n+1)]}.m^{1/(n+1)}.v^{2/(n+1)}$ |
| See section 5.2.2 | $(m/k_2)^{1/n}.A_{max}^{1/n}$ |
| See section 5.3.2 | $m^{4/(4n+1)}.k_2^{-1/n}.k^{1/[n.(4n+1)]}.HIC^{2/(4n+1)}$ |

## 5.2.3 Comments

Gabler et al. (2016) used a finite element (FE) model of the head to compare some quantities only available from a FE simulation with others that are easier to obtain. The quantities of interest included maximum principal strain and maximum linear acceleration. If I understand the implication of their Figure 9 correctly, it is that maximum principal strain is related to $A_{max}$ but not to impact duration (within the range of durations that are of most interest), which agrees with the present analysis. (It is fairly clear that Gabler et al. considered rotational velocity of more importance than linear acceleration.)

Mordaka et al. (2007) used a finite element model of a head impact with a vehicle windscreen. They report (in their Table 5) how a number of output variables changed when impact speed changed by a factor of 4.

- $A_{max}$, HIC, S (maximum deformation of windscreen), and maximum principal strain in the brain respectively changed by factors of 2.4, 33, 4.9, and 6.9.

- Based on section 4.5.1 and Table 4.2, the first three of those factors respectively imply that exponent n is 0.5, 1.0, and 0.7. Based on the result for $x_{2max}$ here, the factor of 6.9 implies n is about 0.4.

According to the theory being used, exponent n should be the same, whichever output variable it is calculated from. I regard the range of variation found, from 0.4 to 1.0, as neither a convincing success nor a convincing failure of the theory.

There are three examples of what are said to be typical force-displacement curves for the human head in Delye et al. (2007), and two more in Monea et al. (2014). They refer to impacts at speeds beteen 2.6 m/sec and 6.9 m/sec. Skull fracture seems to occur at a displacement of between 3.5 mm and 4.5 mm. Displacement is plotted on the horizontal axis, and force on the vertical.

- The relationships are concave upwards. This suggests they might be summarised by a power function having exponent greater than 1.
- Considering the proportionate change in force that accompanies a doubling of displacement from 1.5 mm to 3 mm, this is a factor of between 2.7 and 13 for the five curves. That corresponds to an exponent of between 1.4 and 3.7.

## *5.3 The Skull Fracture Correlate*

### 5.3.1 A contrast of the SFC with HIC

Vander Vorst et al. (2003, 2004) proposed the Skull Fracture Correlate (SFC), measured in an impact test using an instrumented headform, as being a good proxy for the risk of skull fracture. Specifically, they argued that in this respect it is preferable to the Head Injury Criterion (HIC). The SFC is defined to be the average acceleration of the headform over the time period for which HIC is calculated. Consequently it is easy to calculate SFC if HIC is already being calculated. A quantity calculated by DeWeese and Moorcroft (2004) seems to be the same, and there will be some discussion of it in section 11.6; a quantity referred to as effective acceleration by Katagiri et al. (2012, p. 454) also seems to be the same.

Vander Vorst et al. were concerned partly with fracture of post mortem human skulls and partly with testing using an instrumented headform that enables measurement of such quantities as HIC and SFC. In respect of the latter, their central argument lies in the comparison of Figures 8 and 10 in their 2003 paper.

- Strain is found from simulation using a finite element model of the head, and SFC and HIC are calculated from accelerations measured in headform impacts.
- As impact speed changes, Figure 8 shows a very good correlation between strain and SFC, and the data points for targets of different stiffnesses fall on the same line.

- Figure 10 shows a very good correlation between strain and HIC for any particular target that the headform hits, but the lines are different for targets of different stiffnesses.

The starting point of Vander Vorst et al. is that strain is a good proxy for the risk of something breaking. They presume that it is impracticable to measure it in a test, as that would require a headform that was biofidelic as regards skull properties. And their Figure 8 shows that strain can be calculated from SFC. (For their set of impacts, strain was a power function of SFC, being proportional to $SFC^{0.76}$.) There is a similar result in Figures 9 and 10 of the 2004 paper.

Maximum acceleration is not the same as SFC. However, in some circumstances it is related, as follows. Consider acceleration pulses that are all derived from the same basic pulse that is linearly stretched or compressed horizontally (in time) and linearly stretched or compressed vertically (acceleration). For such a set of pulses, SFC is proportional to peak acceleration. It was shown in section 5.2.2 that when m is constant, strain $x_{2max}$ is proportional to $A_{max}^{1/n}$, with $k_1$ and v not entering the equation. Thus in these conditions SFC is a function of strain, again with $k_1$ and v not entering the equation. What Vander Vorst et al. (2003) found in their Figure 8 is, I think, a consequence of this.

## 5.3.2 HIC, strain, and target stiffness

From section 5.2.2, $x_{2max} = (k/k_2)^{1/n}.S$.

From the relationships in section 4.5, S is proportional to $(m/k)^{4/(4n+1)}.HIC^{2/(4n+1)}$.

It follows that $x_{2max}$ is proportional to $(k/k_2)^{1/n}.(m/k)^{4/(4n+1)}.HIC^{2/(4n+1)}$.

That is, $x_{2max}$ is proportional to $m^{4/(4n+1)}.k_2^{-1/n}.k^{1/[n.(4n+1)]}.HIC^{2/(4n+1)}$.

For convenience, the last relationship is shown in Table 5.2.

Vander Vorst et al. are referring to a situation in which m and $k_2$ are constant. The spring constant $k = (k_1^{-1/n} + k_2^{-1/n})^{-n}$, which increases with increasing $k_1$. Therefore the foregoing predicted proportionality relationship may be described as follows.

- Strain is a power function of HIC. The exponent will be 0.4 in the case of n = 1, or 0.29 in the case of n = 1.5.
- The multiplier is an increasing function of target stiffness $k_1$.

Figure 10 of Vander Vorst et al. (2003) is in agreement with that description. Different exponents of HIC are shown for different stiffnesses of target; they are 0.38, 0.38, and 0.31.

### 5.3.3 Interpretation

On the present interpretation, the following points may be made about the SFC.

1. The starting point, that what matters is strain of the skull, does not mention the duration for which the strain lasts. This is highly suggestive of a measure of acceleration alone being required, not something like HIC that takes into account duration.
2. The argument here confirms the empirical finding of Vander Vorst et al. that the SFC is closely related to strain of the skull.
3. However, the SFC is not unique in that property. Other measures of acceleration (e.g., peak acceleration) are, in principle, likely to be as suitable as the SFC as proxies for strain.
4. There may be other reasons for preferring one measure of acceleration rather than another. For example, peak acceleration refers to an instant of time, whereas the SFC is obtained by averaging. Consequently, the SFC may be less affected by random variation.

At present, therefore, I interpret the work of Vander Vorst et al. as follows. If someone thinks that what matters is strain of the skull, but not the duration for which the strain lasts, they probably ought to prefer some measure of average force over most other proxies for injury. Consider the product m.SFC, for example.

- The product m.SFC is preferable to SFC because it is more likely to reflect m correctly.
- Both may be preferable to $A_{max}$ because $A_{max}$ refers to an instant of time, and may be much affected by random variation.
- All three are preferable to HIC because HIC takes into account duration.

However, this assumes that the acceleration pulses under consideration are all the same shape (except for linear stretching or compression on the time axis and linear stretching or compression on the acceleration axis). We might, for example, have in mind one of the standard shapes of pulse, such as a triangle or half a sine wave. Now suppose that changes (e.g., the pulse becomes more peaked or less peaked), in such a way that SFC remains the same. Obviously SFC does not reflect the change. But we might feel the new pulse implies greater injury or lesser injury, in which case we would prefer a proxy for injury that reflected that.

### 5.3.4 Is a compromise between acceleration and force possible?

Someone who is unsure whether acceleration ($A_{max}$) or force ($m.A_{max}$) is the more closely related to injury severity might suggest $m^{0.5}.A_{max}$ as a compromise.

Someone who is sceptical about the proposition that duration does not matter is likely to consider HIC preferable to $A_{max}$.

Starting from HIC, $m^{1.25}$.HIC might be suggested as a compromise between an acceleration-based and a force-based proxy for injury.

## *5.4 Separate consideration of the initial contact*

The differential equations in chapter 4, and the proportionality relationships that were derived, should not be claimed to be suitable for all purposes in all situations. In particular, it might be supposed that some special phenomenon occurs at the initial contact of the colliding objects, that distorts some aspect of what happens.

- The surface of the deforming object may be much less stiff than the remainder. For this, see section 5.4.1 below.
- There may be extra absorption of energy, possibly because the surface is much more stiff than the remainder. For this, see section 5.4.2 below. And it is discussed in the context of penetrating injury in sections 15.6 and chapter 16.
- Some mass of the struck object may be put into motion by the striking object. For this, see section 5.4.3 below.
- When a tube sustains axial impact, maximum force may occur very early, and reflect different aspects of the impact from maximum crush distance. See section 17.14.

### 5.4.1 Maximum displacement

If what is impacted is at first relatively very weak, maximum displacement becomes decoupled from quantities more directly linked to injury such as maximum acceleration and HIC. Thus the proportionality results for maximum acceleration and HIC may be valid in circumstances where those for maximum displacement are not.

Suppose this were thought to be plausible, either because of general knowledge of the surface or because of the pattern of results observed. The theory might be modified by assuming that there is some initial distance c over which acceleration is negligible, and thereafter the differential equation in section 4.3.1 applies. The proportionality relationships (section

4.5 and Table 4.2) will be valid, except that S - c will replace S (for some distance c).

### 5.4.2 Energy absorption at initial contact

It might be assumed that an amount of energy $E_c$ is absorbed at initial contact, and that $E_c$ does not depend on impact speed v or on impactor mass m. (It might depend on other characteristics of the impactor, such as its diameter.) The effective value of v is smaller than the nominal value, and $\sqrt{(v^2 - (2.E_c/m))}$ would replace v. In terms of a critical velocity $v_c$, this could be written as $\sqrt{(v^2 - v_c^2)}$.

It is possible that both energy absorption (as here) and extra displacement (as in section 5.4.1) might be realistic. That is, energy $E_c$ is absorbed over distance c. Two extra parameters is probably too many for most datasets. But giving attention to special phenomena at initial contact almost suggests the possibility of extra experiments to investigate them. And in that context, two extra parameters is not too many.

### 5.4.3 Putting a mass into motion

Suppose a projectile of mass m moving at speed v strikes a stationary mass M, which is unknown and needs to be estimated from data, at the surface of the body struck (e.g., the bonnet), and they then move as one body. (That is, there is no rebound.)

- The mass of that body is m + M.
- Conservation of momentum implies that its speed is (m/(m + M)).v.
- The impact with the remainder of the body struck is now treated as if mass were m + M and speed were (m/(m + M)).v.
- The proportionality relationships will still be valid, but with m + M substituted for m, and (m/(m + M)).v substituted for v.

This will be discussed in section 6.7.4.

Three examples may be mentioned. Ali et al. (2014) considered this in the context of impacts with built structures. They described it as a method of taking into account inertial resistance of the target. Such a model has been used in the context of cracking of eggs: see section 18.12. Morye et al. (2000) considered energy absorption by putting mass into motion in the context of penetration of polymer composites by ballistic impact. However this was rather different, as there was not a hypothesis of a specific mass and then application of conservation of momentum.

## *5.5 Discussion*

### 5.5.1 Practicability of more realistic models

Section 5.4 considered some options for extending the proportionality results that are derived from the Hunt and Crossley equation. Those options might be considered inelegant add-ons to the core theory. Instead, is it likely that a more realistic equation or model could be used to obtain analogous results?

Yes, I think that is quite likely. The Hunt and Crossley equation happens to be one that I have heard about, and from which some results can be obtained, but there are other possibilities. I have in mind a paper by Shivakumar et al (1983).

### 5.5.2 Possible relevance of theories for penetrating impacts

It is possible that theories appearing in the literature of penetrating injury may have some relevance to blunt impacts. See section 15.8.

### 5.5.3 Application to results obtained across various impact locations

Chapters 4 and 5 have concentrated on the effects of varying v, m, and perhaps d at a given impact location. Some of the findings may be adapted to datasets of results obtained at a number of different impact locations.

The motivation for this is that routine pedestrian headform testing is conducted with a specified headform and at a specified speed: m and d are constant, and v is nearly constant (it varies a little because of variation in bonnet angle). In addition, there is usually no measurement of k (static or dynamic) available. Thus there may exist quite extensive datasets for which the impact surface varies and a number of output variables are observed.

In these circumstances, it may be interesting and practicable to examine the co-variation of the output variables: in particular, $A_{max}$, HIC, and S. For this, see sections 6.5 - 6.7.

# 6. What theory suggests about data description and analysis

I am interested in data, and seeing what theoretical ideas it supports and what it tends to refute. Looking at data is also a good method of *generating* ideas. It is quite common to test things that may hit a human, or that are intended to protect the human. The analysis of data collected in such tests seems to me to be often incomplete. In many chapters of this book (especially chapters 7 - 14, 16 - 18), I will comment on published data.

I should emphasise that I do not intend to criticise the authors whose work I examine.

- On the contrary, it is praiseworthy that they have published their work with sufficient clarity that it can be fruitfully discussed.
- Impact experimentation is difficult. I am not a practical experimenter, and I hope I am usually sympathetic about the problems faced.
- There has not been much theory available, and thus it has been challenging to get the most out of data.

## *6.1 Consequences of a theoretical model*

On the basis of section 4.5, it is worthwhile investigating whether the relationship between each of several independent (input) variables and each of several dependent (output) variables is a power function.

- Input variables: speed of impact, mass of headform, surface impacted.
- Output variables: maximum deformation, maximum acceleration, HIC. (Duration of impact is not often analysed. Also, if n is close to 1, duration will be almost independent of impact speed.)

And the exponents in the relationships should all be consistent with some particular value of the exponent n.

Concerning surface impacted, there is usually no quantitative measure available (e.g., stiffness). Instead, it might simply be recorded as impact point 1, 2, 3, etc. Thus analysis will need to be of the co-variation of output (dependent) variables, rather than of the dependence of output variables on stiffness --- see sections 6.5 - 6.7 especially.

## *6.2 Empirical studies*

Blunt injury is important in many contexts --- transport, at work, at home, playing sports, children's play, military conflict, and so on. Blunt damage (especially to manufactured objects, and to agricultural produce) is important also.

There is therefore quite a lot of relevant experimentation. Some impressions I have of the literature (including experiments conducted outside of the road accident context) are as follows.

- Data analysis as suggested in this and later chapters is not often carried out.
- Relationships are usually in the expected directions.
- Relationships are usually compatible with power functions.
- However, the consistency of the experimentation and the accuracy of measurement are often not good enough to say whether relationships are power functions or whether some other function is a better description.

I have occasionally seen papers in which the results are complicated and puzzling, when viewed in the light of the predictions in Hutchinson (2013) and section 4.5 above. It might be said that such papers constitute evidence that the predictions are poor ones. But I am not convinced of that. It seems more likely to me that something went wrong (and there are generally so few data points that even one false result is likely to obscure the whole pattern), or that some phenomenon occurred that is outside the scope of the theory (e.g., bottoming out).

## *6.3 Treatment of data*

### 6.3.1 Linearising a power function by taking logarithms

A power relationship, such as those in section 4.5, implies a straight line after taking logarithms of the variables. If $A_{max} \propto v^{2n/(n+1)}$, for example, $\ln(A_{max})$ will be linearly dependent on $\ln(v)$, the slope being $2.n/(n + 1)$.

Consequently, the data is treated as follows. Take logarithms of the independent variables (such as v and m), take logarithms of the dependent variables (such as $A_{max}$, HIC, and S), prepare scatterplots, examine whether these show good straight-line relationships or not, and (if so) fit straight-line relationships using regression.

Estimates of the exponent n can be obtained from the estimates of the slopes. If the slope of $\ln(A_{max})$ versus $\ln(v)$ is estimated to be p, for example, then from $p = 2.n/(n + 1)$ the exponent n may be estimated as $p/(2 - p)$.

Checking whether a plot of the logarithms is approximately a staight line --- perhaps you think this is too elementary to be worth your attention?

- Yes, it is elementary.

- It does help understanding. Better understanding usually helps fix results in your memory, and often leads to new questions and new answers.
- When exptrapolating, a power function does give results that are slightly different from those of a straight line.
- But the chief benefit is probably in what the several proportionality results of section 4.5.1 suggest about other variables. That is, having found an approximate straight line when plotting the logarithms of one pair of variables, the next time you do a similar experiment, you will consider measuring one or two extra dependent variables, and verying one or two extra independent variables. Thus you will observe multiple relationships, and be able to assess their consistency with those proposed in section 4.5.1.

### 6.3.2 Drop height instead of impact speed

Quite a popular method of experimentation is to drop something on to something else. For example, an instrumented headform might be dropped on to a foam material to test the foam's energy-absorbing properties and its suitability for use in protecting humans from impacts.

Experimenters often consider that there is a very tight relationship between drop height and impact speed, for example, $v^2 = 2.g.h$, where h is the drop height and g is acceleration due to gravity. That particular relationship assumes there is no resistance due to air or due to friction with a guiding rail. Consequently, results are often reported with h as the independent variable, not v.

The proportionality relationships may be written in terms of h by replacing v with $h^{0.5}$.

$A_{max} \propto h^{n/(n+1)}$

$HIC \propto h^{(4n+1)/(2n+2)}$

$S \propto h^{1/(n+1)}$

Thus, with appropriate modifications to the interpretation of the results, ln(h) may be used in the scatterplots and regressions in place of ln(v).

## 6.4 Simple reasons why an expected relationship might not appear

*Maximum displacement might be misleading*. If what is impacted is at first relatively very weak, maximum displacement becomes decoupled from quantities more directly linked to injury such as maximum acceleration and HIC. Thus the proportionality results for maximum acceleration and HIC may be valid in circumstances where those for maximum displacement are not. This is discussed in section 5.4.1.

*Impact speed might be inaccurate*. If this is so, all relationships between the dependent variables and v will be distorted. One type of reason for an inaccurate impact speed is assuming that it is proportional to the square root of drop height when that is not so.

*An extra phenomenon at initial contact*. For extra energy absorption at the moment of initial impact, see sections 5.4.2 and 15.6. For putting a mass into motion, see sections 5.4.3 and 6.7.4. For axial impact of a tube, see section 17.14.

*Instrumentation*. I have generally not questioned the experimental techniques (for example, the methods of measuring acceleration, or force, or speed, or displacement) or the filtering (smoothing) of the acceleration trace.

## 6.5 How pairs of dependent variables co-vary: HIC and maximum acceleration

### 6.5.1 Data description and interpretation

In addition to relationships between the independent variables and the dependent variables, the way that two dependent variables co-vary may be of interest. This is especially the case for maximum acceleration and HIC, as these are both used to reflect injury. And either of these might be plotted versus maximum displacement, also (section 6.6).

There may be particular reasons for plotting one dependent variable versus another.

- An independent variable might be absent. For example, there might be a categorical variable identifying impact location, but no quantitative variable giving the surface stiffness.
- There may be some doubt about the validity or the accuracy of an independent variable.
- An independent variable might not be appropriate in the context of some theory of interest. For example, it is inappropriate to use actual impact speed as the independent variable if it is believed

that an effective impact speed is the relevant variable, as mentioned in section 5.4.2.

Experiments are often performed in which impact speed v is varied, and maximum acceleration $A_{max}$ and the Head Injury Criterion HIC are measured. There are then three estimates of the exponent n: from the dependence of $A_{max}$ on v, from the dependence of HIC on v, and from the dependence of HIC on $A_{max}$. It may happen that the third of these is quite different from the other two. For a possible reason for this, see Appendix 5.

### 6.5.2 Reasons for co-variation

The relationships obtained in section 4.5 throw some light on the question of whether HIC and maximum acceleration $A_{max}$ --- which are both intended to reflect the likely severity of injury --- are equivalent concepts. In a sense, of course, it is obvious that they are not, as it is not possible to calculate one from the other. Nevertheless, they are to some extent similar, and the following will clarify the relationships between them.

Suppose we measure both HIC and $A_{max}$ in a number of experiments. We might find that HIC and $A_{max}$ are highly correlated, and we might be able to obtain an empirical formula for calculating one from the other. However, the relationships obtained in section 4.5 imply that this relationship will differ according to what the source of the correlation is. Is it, for example, due to variation in v, or m, or k?

Sections 6.5.3 and 6.5.4 will rephrase, in several ways, the question of whether HIC and $A_{max}$ are more or less equivalent.

### 6.5.3 Co-variation due to variation in one factor (speed, headform mass, or surface stiffness)

Are HIC and $A_{max}$ sufficiently closely linked that, if we are referring to a particular impact point on a particular vehicle (this refers to a particular stiffness, among other things) and a particular headform, then as speed varies, HIC and $A_{max}$ co-vary and it is possible to calculate one from the other?

- A proportionality relationship may be obtained.
- The relationships that hold for different impact points and different headforms are likely to be different.
- There may be something similar in the relationships that hold for different impact points and different headforms. For example, after taking logarithms, the slope may be the same.

Are HIC and $A_{max}$ sufficiently closely linked that, if we are referring to a particular impact point on a particular vehicle and a particular speed, then as headform mass varies, HIC and $A_{max}$ co-vary and it is possible to calculate one from the other?

- A proportionality relationship may be obtained.
- The relationships that hold for different impact points and different speeds are likely to be different.
- There may be something similar in the relationships that hold for different impact points and different speeds. For example, after taking logarithms, the slope may be the same.

Are HIC and $A_{max}$ sufficiently closely linked that, if we are referring to a particular speed and a particular headform, then as impact point (and thus presumably stiffness) varies, HIC and $A_{max}$ co-vary and it is possible to calculate one from the other?

- A proportionality relationship may be obtained.
- The relationships that hold for different speeds and different headforms are likely to be different.
- There may be something similar in the relationships that hold for different speeds and different headforms. For example, after taking logarithms, the slope may be the same.

### 6.5.4 Co-variation due to variation in two factors

Consider now a particular impact point on a particular vehicle (for example), and variation in both speed and headform mass. Are HIC and $A_{max}$ sufficiently closely linked that as speed and headform mass vary, HIC and $A_{max}$ co-vary and it is possible to calculate one from the other?

Now the answer is no. One pair of conditions may have the higher HIC and the other have the higher $A_{max}$.

As a numerical example, suppose that n = 1, and that therefore the exponents of m are -0.5 (for $A_{max}$) and -0.75 (for HIC), and the exponents of v are 1 (for $A_{max}$) and 2.5 (for HIC). Suppose m is multiplied by 2 and v is multiplied by 1.27. The effect on $A_{max}$ is to multiply it by $2^{-0.5}.1.27$, and the effect on HIC is to multiply it by $2^{-0.75}.1.27^{2.5}$. That is, the effect on $A_{max}$ is to reduce it by 10 per cent, and the effect on HIC is to increase it by 8 per cent.

### 6.5.5 Proportionality relationships

The following relationships were obtained in section 4.5 and Table 4.2.

$$A_{max} \propto (m/k)^{-1/(n+1)}.v^{2n/(n+1)}$$

$HIC \propto (m/k)^{-1.5/(n+1)}.v^{(4n+1)/(n+1)}$.

HIC is thus proportional to $A_{max}^{1.5}.v$, irrespective of m and k. (This relationship will be considered in sections 7.8.3, 9.3.3, 10.10.4, and 14.3.3.)

If m and k are constant and v is varying, $HIC^{2n/(4n+1)}$ is proportional to $v^{2n/(n+1)}$, and therefore $HIC^{2n/(4n+1)}$ is proportional to $A_{max}$. Thus HIC and $A_{max}$ are connected by a power relationship, but the exponent is not known. (It might be thought that n is approximately 1. Then $HIC^{0.4}$ would be proportional to $A_{max}$. That is, HIC would be proportional to $A_{max}^{2.5}$).

Similarly, if k and v are constant and m is varying, $HIC^{2/3}$ is proportional to $A_{max}$ (that is, HIC is proportional to $A_{max}^{1.5}$). Thus HIC and $A_{max}$ are connected by a power relationship, and the exponent is known (and does not depend on n).

And if m and v are constant and k is varying, $HIC^{2/3}$ is proportional to $A_{max}$ (that is, HIC is proportional to $A_{max}^{1.5}$). Thus HIC and $A_{max}$ are connected by a power relationship, and the exponent is known (and does not depend on n).

In short, the relationship between HIC and $A_{max}$ will depend on what it is that is responsible for the co-variation between them. In a study of motorcycle helmet drop-test performance, Zellmer (1993) considered the possibilities that HIC is proportional to $A_{max}^{2.5}$, or (alternatively) HIC is proportional to $A_{max}^{1.5}$. He did not quite appreciate that either might occur, depending on what the source of the co-variation is.

For many years, some people have thought that when v changes, HIC is proportional to $A_{max}^{2.5}$. They are correct, if n is 1.

- It is not clear to me exactly what people have thought, or on what grounds. The belief has probably arisen from algebra that shows this to be the case in some special cases of pulse shape, provided pulse shape does not change when v changes (Chou and Nyquist, 1974), and appreciation that pulse duration does not change in the case of the linear spring.
- HIC is proportional to $A_{max}^{(4n+1)/(2n)}$, and the exponent must be greater than 2, as n is positive. It is more flexible than the idea that $HIC \propto A_{max}^{2.5}$, which has some history behind it.

## *6.6 How pairs of dependent variables co-vary: Maximum deformation and a proxy for injury severity*

Now consider the correlation between a proxy for severity of injury to the head (such as HIC or $A_{max}$) and a measure of amount of damage to the vehicle (such as S). This will provide another example of considering the source of (that is, the reason for) the correlation.

*Impacts with a particular surface at different speeds*. Here, speed is the reason for the correlation. For a given surface, increasing speed will mean increasing HIC (i.e., increasing severity of injury) and increasing maximum deformation (i.e., increasing damage to the vehicle). There will be positive correlation between the two output variables.

*Impacts with various surfaces at a particular speed*. Here, the variation in the surface is the reason for the correlation. The simplest type of variation in an impacted surface is in respect of its stiffness. For a given speed, increasing stiffness will mean increasing HIC (i.e., increasing severity of injury) and decreasing maximum deformation (i.e., decreasing damage to the vehicle). There will be negative correlation between the two output variables.

This example is dramatic because changing the source of the correlation changes the co-variation from positive to negative. In the case of the co-variation of HIC and maximum acceleration discussed in section 6.5, the relationship changed quantitatively, but remained positive.

## *6.7 How pairs of dependent variables co-vary: Empirical relationships that contradict theory*

### 6.7.1 The problem

Suppose that we attempt to predict ln(HIC) from ln($A_{max}$), with impact speed being the source of the co-variation, and we find the relationship is approximately a straight line. The slope is (4.n + 1)/(2.n), which is always greater than 2. What conclusion should we draw if the slope is found to be less than 2?

Or, having estimated n, is it similar to the estimate from the dependence of ln(HIC) on ln(v), and to that from the dependence of ln($A_{max}$) on ln(v)?

Suppose that we attempt to predict ln(HIC) from ln($A_{max}$), with impact surface being the source of the co-variation, and we find the relationship is approximately a straight line. The slope is predicted to be 1.5. What conclusion should we draw if the slope is found to be quite different from this?

Similar questions arise if the relationship between ln(S) and either ln(HIC) or ln($A_{max}$) is not according to theory.

### 6.7.2 General replies

Faced with such questions, there are a number of general replies that could be made.

- The observed value might not statistically significantly different from the theoretical value.
- If the difference is statistically significant, there might be some reason to disbelieve the standard error, and thus to disbelieve the statistical significance.
- There may be some reason to disbelieve some of the data points. Some of them may appear to be outliers, for example.
- Something that is supposed to be constant (such as impact speed, impact angle, or impact surface) may not be constant.
- There may be some reason to disbelieve one or other of the variables being plotted. I have in mind that sometimes there may be scepticism about $A_{max}$, which refers to only a very brief instant of time, and is sensitive to the filtering of the acceleration pulse. Or it may be felt that v ought to be replaced with an effective speed, as suggested in section 5.4.2.

Another response is to say the theoretical relationships are obtained from the differential equation in section 4.2.5, and that there is no reason to think that this equation will be followed for all speeds and for all impact surfaces. Failure of the prediction means that the differential equation was not obeyed in the experiment concerned. The problem with this response is that it is vague.

Sounik et al. (1997) report some data showing opposite behaviour of $A_{max}$ and HIC. In the case of a polyurethane foam that they refer to as System B, an increase of its density led to a slight decrease in $A_{max}$ but a slight increase in HIC. That is illustrated in Figures 8 and 9 of Sounik et al. There is nothing in chapter 4 that implies a foam should change its energy-absorbing properties in a simple way when its density changes; it might have been the case that the effect was simply a change of k, with HIC thus being proportional to $A_{max}^{1.5}$ (as in section 6.5.5), but apparently that was not so.

### 6.7.3 A reply giving priority to the relationship between $A_{max}$ and HIC

A more specific reply is as follows. Unless there is scepticism about $A_{max}$, the relationship between HIC and $A_{max}$ is the one there is least doubt about.

- Both HIC and $A_{max}$ are calculated from the period when acceleration is high.
- The relationship between HIC and $A_{max}$ does not rely on v being correct, or on S not being distorted by there being very low stiffness or very high stiffness at initial contact.

Suppose that n is obtained from the relationship between HIC and $A_{max}$, or is assumed to be 1 (linear spring), or is assumed to be 1.5 (Hertzian impact). It may then be possible to ask if this value of n is consistent with the empirical relationship between ln(HIC) and ln(v), and with that between ln($A_{max}$) and ln(v).

If it is not consistent, it may be possible to calculate effective speeds that would replace actual impact speeds and rescue the theory. A straightforward way of using an effective speed is to suggest it arises from a constant amount of energy being absorbed at initial contact.

- The reduction from actual impact speed to effective impact speed will be proportionately great at low speeds and negligible at high speeds.
- When plotting ln($A_{max}$) and ln(HIC) versus ln(v), the slopes will be smaller than otherwise.
- Consequently the estimate of the exponent n will be smaller than otherwise.

If, on the other hand, the n's estimated from ln($A_{max}$) and ln(HIC) versus ln(v) are fairly large but n estimated from ln(HIC) versus ln($A_{max}$) is smaller, an explanation in terms of an effective impact speed is unlikely to be successful.

Unfortunately, while a principled argument can be made, as above, that the relationship between HIC and $A_{max}$ is the one there is least doubt about, in practice a lot of random error can enter, as is shown in Appendix 5.

### 6.7.4 Putting a mass into motion

It seems possible to me that the following may justify alternative relationships between $A_{max}$ and HIC.

As in section 5.4.3, suppose a projectile of mass m moving at speed v strikes a stationary mass M at the surface of the body struck (e.g., the bonnet), and they then move as one body. (That is, there is no rebound.) Mass M is presumed not to depend on v; it might, perhaps, depend on the diameter of the impactor.

- The mass of that body is m + M.
- Conservation of momentum implies that its speed is (m/(m + M)).v.
- The impact with the remainder of the body struck is now treated as if mass were m + M and speed were (m/(m + M)).v.
- The proportionality relationships will still be valid, but with m + M substituted for m, and (m/(m + M)).v substituted for v.
- Thus the exponent of v will be unchanged in the proportionality relationships. The ways that the various outputs depend on m will change.
- There will be an application of this at the end of section 18.12.

From the proportionality relationships given in section 4.5, a little algebra shows that the ratio $(A_{max}^{1.5}.v)/HIC$ is proportional to 1 + (M/m).

- That is, there is a straight-line relationship between $(A_{max}^{1.5}.v)/HIC$ and the reciprocal of mass m.
- Consequently, it is possible to estimate M from observations of the ratio at two masses.
- Let $Q = (A_{max}^{1.5}.v)/HIC$. Suppose this is $Q_1$ for mass $m_1$, and $Q_2$ for mass $m_2$. Then $M = [(Q_1 - Q_2) / (Q_2 - (m_1/m_2).Q_1)].m_1$.

# Part B: Re-examination of published data

Blunt trauma may occur in many circumstances, such as road accidents, workplace accidents, recreational accidents, military and security operations, and so on. Experiments are conducted based in many different disciplines, such as the specific contexts already named, mechanical engineering, materials science, medicine, and so on.

That much is uncontroversial. What may be controversial is that many experimenters do not get as much as they might out of their data. Possible reasons are a focus on immediate practical matters (and failure to imagine other potential uses of their data), and lack of theory to help interpret the data. The present book advocates the employment of theory, and (as proposed in chapter 6) methods of data analysis that are compatible with theory.

Chapters 7 - 14 give examples from several contexts of application. Summary of the results using power functions and interpretion with the exponent n from the Hunt and Crossley equation, as discussed in chapters 4 and 5, is by me (and the original authors should not be blamed if I have blundered). In some cases I have read data from scatterplots, in which case small errors are bound to be present. The examples are not the result of systematic search in the research literature, but are simply ones that I have come across from time to time.

I think these examples demonstrate the helpfulness of theory in interpreting data. There will be discussion of this at the end of chapter 7 (sections 7.13 and 7.14), and in chapter 20.

Part B has the following chapters.

7. Data from impact tests, 1: Transport contexts.
8. Data from impact tests, 2: Sports (with or without helmets and other protection).
9. Data from impact tests, 3: Sports (ground impact).
10. Data from impact tests, 4: Children's play (ground impact).
11. Data from impact tests, 5: Adults.
12. Data from impact tests, 6: Military and security contexts.
13. Data from impact tests, 7: The Viscous Criterion.
14. Data from impact tests, 8: Rigid surfaces.

It is understood here that the impact tests are relevant to human injury. Most of the examples (though not those in chapter 13) are concerned with head injury, acceleration being the crucial quantity. As will be said at the beginning of Part C, Chapters 15 and 16 will be on penetrating injury, and then chapters 17 and 18 will discuss impact tests relevant to damage of things.

# 7. Data from impact tests, 1: Transport contexts

## *7.1 Introduction*

What can you expect from this chapter and others that re-examine published data?

- A few people reading the chapter will be seriously interested in the examples discussed. They will read the paper referred to, understand the experiment and the data presented, and evaluate the suggestions and comments that I make.
- But I am not expecting that most people will be so deeply interested. Most will not read the paper referred to. So I am intending that you will get value simply from the brief account that I give. You will gain an idea of what questions are studied, the types of experiment performed, the amount and nature of the data collected, and the associated limitations. You will be able to appreciate that the experiments are usually quite narrowly focused, with little concern for theory or for experimental conditions other than those actually used.
- Some readers will be much more interested in one area of application than in others. I am not concerned, then, that there is a good deal of similarity between the examples in chapters 7, 8, 9, 10, and 11. Many people will appreciate seeing a range of examples from their specific area.
- Other readers, perhaps, will feel great interest in unfamiliar contexts of application.
- The comments that I make about the experimental results are usually quite simple ones, and I have not supported them with graphs and scatterplots and complicated statistics.

My reaction to the examples in sections 7.2 - 7.12, and I expect many readers will share it, is that extra insights into a dataset can be obtained by comparing it with theory, but these insights are rather limited, and it seems that theory can help only a little with present-day datasets. There will be further comments in sections 7.13 and 7.14, and in chapter 20. Chapter 19 of Hutchinson (2018) is similar to this chapter.

Concerning road safety, some examples in chapters 8 - 14 are also relevant, as blunt injury from other causes is often similar to that in road accidents. In addition, section 17.9 discusses damage to cars in impacts, and section 17.14 the crushing of tubes (which might be used as parts of the structure of vehicles).

As noted in section 6.1, it is worthwhile investigating whether the relationship between each of several independent (input) variables and each of several dependent (output) variables is a power function. The exponents in the several relationships should all be consistent with some particular value of the exponent n appearing in the differential equation (section 4.3). There will also be some analysis of the co-variation of output variables.

At times, two or more exponents will be discussed almost at the same time. Do not get them muddled.

- For example, a relationship given in section 4.5.1 is that S is a power function of v, the exponent being 2/(n + 1). But n is the exponent in the power-function dependence of force on displacement (see the differential equations in sections 4.2.4, 4.2.5, and 4.3.1).
- Thus I will need to use language such as: the exponent in the dependence of S on v is estimated to be 0.5, which implies that the exponent n is about 3.

## *7.2 Searson et al. (2012a)*

Searson et al. (2012a) reported on 29 headform impact tests at several speeds on the exterior of passenger vehicles.

- Impacts were made at seven locations on vehicles. These were on three different models of vehicle.
- At least three speeds of impact were used at each location. The lowest speed was approximately 75 per cent of the highest.
- Both HIC and maximum deformation were reported.

Searson et al. gave some theory for the dependence of HIC on impact speed and the dependence of maximum deformation on impact speed. The theory was based on assuming the impact was with a linear spring. On that assumption, they found the two dependences would be power functions with slopes of 2.5 and 1.0. Thus the relationships were expected to be linear on logarithmic axes, with slopes of approximately 2.5 for HIC and 1.0 for maximum deformation.

Findings may be summarised as follows.

- Observed relationships were approximately straight lines after logarithmic transformation: this means that the dependence of HIC on impact speed and the dependence of maximum deformation on impact speed are approximately power functions.
- Those conclusions can only be tentative. A stronger conclusion is impossible because only a few speeds were used at each location, and only a narrow range of speeds was used at each location. The data is compatible with other functional forms as well as with power functions.
- For HIC, the slopes were between 1.61 and 3.04 for the seven locations, averaging about 2.5.

- For maximum deformation, the slopes were between 0.65 and 1.33 for the seven locations, averaging about 0.8.
- There was a negative correlation between the two slopes. (The location having the highest slope for dependence of HIC on speed had the lowest slope for dependence of maximum deformation on speed, and vice versa.)

The theory of section 4.3 has the linear spring as a special case, but is more general.

- A slope other than 2.5 for HIC and other than 1.0 for maximum deformation is interpreted as the exponent n in the differential equation (section 4.3) being different from 1.
- From section 4.5, the two slopes will be (4.n + 1)/(n + 1) for HIC and 2/(n + 1) for S. Let these be slopeH and slopeS. Eliminating n, slopeH is predicted to be (8 - 3.slopeS)/(3 - slopeS). For slopeS being between 0.6 and 1.4, slopeH would be predicted to be between 2.58 and 2.37.
- Thus a negative relationship between the two slopes is predicted.
- However, the predicted relationship is rather different from that found. The empirical result was that as slopeS changed from 0.65 to 1.33, slopeH changed from 3.04 to 1.61.
- The wide range of slopes --- implying a wide range of n --- is itself to some extent a point against the theory. There were similar impacts on similar objects: one might expect the slopes to be similar.
- Searson et al. said that there was no bottoming out in their tests. Nevertheless, it may be pointed out that bottoming out in the impact at the highest speed would imply low S and high HIC, and thus low slopeS and high slopeH.

## *7.3 Mizuno and Kajzer (2000)*

The source of the data is Figure 23 of Mizuno and Kajzer (2000) (also, Figure 9 of Mizuno et al., 2001). That Figure plots HIC at three impact speeds, for one location on a bonnet and one location on a windscreen.

The data may be summarised as follows. (All figures are approximate.)

- Bonnet. When impact speed changes by a factor of 1.67, HIC changes by a factor of 3.24. The exponent for the dependence of HIC on speed is therefore 2.30. This corresponds to n being about 0.8.
- Windscreen. When impact speed changes by a factor of 1.67, HIC changes by a factor of 6.03. The exponent for the dependence of HIC on speed is therefore 3.52. This corresponds to n being about 5.0.

Mizuno and Kajzer did not interpret the two relationships as power functions, but referred to them as linear. (HIC would be zero when speed is zero. A power function predicts this correctly, but a straight line does not.)

Pereira (2010) reports HIC at eight bonnet locations, each at two speeds (35 km/h and 40 km/h). A child headform was used at five locations, and an adult headform at three. The factors by which HIC increased at the higher speed imply exponents of between 1.3 and 3.0 for the dependence of HIC on v (if dependence is assumed to be a power function). These fall within the range of 1 to 4 that is implied by the relationships of section 4.5.1.

For another aspect of the work of Mizuno and Kajzer (2000) (also, Mizuno et al., 2001), see section 7.5.

There is a little data on both $A_{max}$ and HIC versus impact speed in Mizuno et al. (2017). It refers to A-pillar impacts of helmeted pedal cyclists.

## *7.4 Anderson et al. (2000)*

Anderson et al. (2000) reported on tests in which a headform was dropped on to a specimen of a material under test, that was supported on a steel slab. The context of this work was the development of a protective headband intended for wearing by car occupants. Results for impacts at 16.0 km/h and 19.2 km/h with five materials are given. The five materials were referred to as: CF-45100 at a temperature of 15-17 degrees, BB-38 at 25-26 degrees, E175 at 25-26 degrees, E900_5.6 at 25-26 degrees, and E900_6.0 at 25-26 degrees.

It is convenient to give the relevant theoretical results first, and then examine the empirical findings. The higher speed of impact is 1.20 multiplied by the lower speed. For the Hunt and Crossley model, section 4.5 gave some theoretical results.

- The exponent for maximum acceleration is 2.n/(n + 1), which cannot exceed 2. Thus the ratio of maximum acceleration for the two speeds cannot exceed $1.2^2 = 1.44$.
- The exponent for HIC is (4.c + 1)/(c + 1), which cannot exceed 4. Thus the ratio of HIC for the two speeds cannot exceed $1.2^4 = 2.07$.

The findings in Anderson et al. (2000) were as follows.

- For maximum acceleration, the prediction was disconfirmed for all five materials. The median ratio of maximum acceleration at the two speeds was 1.68.
- For HIC, the prediction was disconfirmed for four of the five materials. The median ratio of HIC at the two speeds was 2.52.

That is, the effects of speed on maximum acceleration and HIC are greater than is possible with the Hunt and Crossley model.

A possible reason for inapplicability of this model is bottoming out. For CF-45100 at 15-17 degrees, this is discussed by Anderson et al. (p. 14 of their report).

## *7.5 HIC and maximum deformation*

The situation considered here and in the next section is that the object impacted, or the impact location on one object, is changing, and speed is not. Such data may be quite common. Many impact tests that are reported are all conducted according to the same protocol: conditions are identical, including impact speed, but the impacts are on different objects.

HIC is proportional to $S^{-1.5}$ if impact speed, b, and n are constant (see section 4.5.3).

Relevant results include the following.

- Figure 24 of MacLaughlin and Kessler (1990) (also, Figure 7 of NHTSA, 1993) shows a strong negative relationship between HIC and S, for approximately 14 impact tests on the bonnets of each of 12 vehicles. The impacts were at 23 mile/h, normal to the surface. A power curve fit is shown, HIC $\propto S^{-2.1}$. MacLaughlin and Kessler note that several of the high values of HIC probably occurred because of contact with the suspension system tower; such bottoming out would be expected to distort the relationship given in section 4.5.3.
- Figure 25 of Mizuno and Kajzer (2000) shows strong negative relationships between HIC and S, for a set of tests on the bonnet top and a set of tests on the windscreen of a car. The impact speed was 40 km/h. Power curves are shown, with exponents of -2.1 (bonnet) and -1.8 (windscreen). Another account is in Mizuno et al. (2001).
- Figure 13 of Han and Lee (2003) shows a strong negative relationship between HIC and S, for 13 impact tests on the bonnet of a production car and one modified for greater pedestrian safety. The impacts were at 40 km/h, with a 2.5 kg headform.

For the data of Han and Lee (2003), I have plotted ln(HIC) vs. ln(S).

- The relationship is a good straight line.
- The relationship appears to be the same for the modified car as for the production car.
- The slope is about -2.2.

Naturally, the data points differ in respect of location on the car bonnet. If locations differed only in respect of stiffness (and the Hunt and Crossley model of chapter 4 were valid), the slope would be -1.5. But it is likely the headform was projected at a specific angle to the horizontal, and as the bonnet is at different angles to the horizontal at different locations, the component of velocity normal to the bonnet surface would be different at different locations. This might account for the relationship being rather different from that expected.

Masoumi et al. (2011) conducted finite element modelling of pedestrian headform impacts with bonnets. Both HIC and S are reported for 8 locations on 3 bonnets (steel standard design, aluminium standard design, and composite simplified design). I have plotted ln(HIC) vs. ln(S) for the three bonnets.

- The relationship is a good straight line. The slope is estimated to be about -1.1. (This is not significantly different from -1.5.)
- Some choices made in obtaining that result should be noted. The modelling included some adult headform impacts and some child headform impacts. The headforms differed in respect of mass, diameter, and angle at which they were projected, but I did not disaggregate the analysis. In the case of four impacts, S (but not HIC) is given as exactly the same. I have included the first of these, as the acceleration-displacement curve (Figure 11 of Masoumi et al.) confirms the result, but excluded the others.

There are some further results from finite element modelling in Shojaeefard et al. (2014).

## *7.6 Miller et al. (1996)*

Section 6.5.5 considered what the relationship between HIC and $A_{max}$ would be if variation in stiffness k were responsible for their co-variation. HIC would be proportional to $A_{max}^{1.5}$.

Miller et al. (1996) plotted HIC(d) (a modified version of HIC) versus $A_{max}$ for approximately 400 Free Motion Headform (FMH) impacts with vehicle interiors, the impact speed being 15 mile/h (perpendicular to the surface). Their Figure 4 shows a strong positive correlation between HIC(d) and $A_{max}$. Miller et al. had no theory for the relationship, and they fitted an empirical quadratic equation to the dependence of HIC(d) on $A_{max}$. The scatterplot can be seen to be concave upwards, and this is confirmed by the coefficient of the term in $A_{max}^2$ being positive.

The original HIC can readily be calculated from any given HIC(d). Looking at Figure 4 of Miller et al., at $A_{max}$ = 100, HIC(d) is seen to be approximately 500, and this implies HIC = 442; at $A_{max}$ = 300, HIC(d) is approximately 2000, and this implies HIC = 2430.

Thus multiplying $A_{max}$ by 3 corresponds to multiplying HIC by 5.5 (= 2430/442). Consequently, HIC is approximately proportional to $A_{max}^{1.55}$ (as $3^{1.55}$ is 5.5). This is close to the predicted relationship that HIC is proportional to $A_{max}^{1.5}$.

## 7.7 Oikawa et al. (2016)

Oikawa et al. (2016) report both maximum acceleration and HIC for impacts of a headform, either wearing or not wearing a bicycle helmet, with the A pillar of a vehicle. They also report both maximum acceleration and HIC for impacts of a headform, either wearing or not wearing a bicycle helmet, with the road pavement. This data is interesting particularly because the values of HIC were high: over 2000 in the A-pillar impact of a headform with helmet, and over 800 in the road surface impact of a headform with helmet. For the A-pillar impact, the impact speed was 35 km/h (but this was not at 90 degrees to the A pillar). For the road surface impact, the drop height was 1.5 m, and the speed was thus approximately 20 km/h.

If the effect of a helmet is to change the stiffness parameter k in the Hunt and Crossley equation, HIC would be proportional to $A_{max}^{1.5}$ (see section 6.5.5).

- For the A-pillar impact, the empirical results of Oikawa et al. imply an exponent of 1.48.
- For the road surface impact, the empirical results imply an exponent of 1.42.

Ito et al. (2014) report a finite element simulation of a cyclist's impact with an A pillar at 41.8 km/h. A bicycle helmet did reduce HIC and $A_{max}$, but by only 21 per cent and 6 per cent respectively. Ito et al. note that substantial bottoming out of the helmet liner occurred. The percentage changes are in the ratio 3.5, not 1.5; thus in this case, wearing a helmet seems to be not equivalent to a change of stiffness. Mizuno et al. (2017) also report a finite element simulation.

For other studies directly concerned with bicycle helmets, see sections 14.3 and 14.4.

## 7.8 Monk and Sullivan (1986)

### 7.8.1 Introduction

Monk and Sullivan (1986) were concerned with padding a vehicle's A pillar to mitigate the impact of an occupant of the vehicle. They were aware that Chou and Nyquist (1974) had established that HIC = $0.0296.deltaV.A_{max}^{1.5}$ for a half sine acceleration pulse. Monk and Sullivan felt that velocity change deltaV was probably close to impact speed v in the tests they considered. These were of 1 inch thick padding, impact speeds being between 5 mile/h and 20 mile/h. Monk and Sullivan found that values of HIC predicted on that basis were within 15 per cent of those observed. The tests used a Hybrid III headform.

### 7.8.2 Effects of impact speed

Concerning the effect of impact speed, I have fitted three equations to data in Table 5 of Monk and Sullivan. In each case, a constant effect of padding material was also estimated. Thus in each case the slope represents an average over all materials.

- In a regression of ln(S) in terms of ln(v), a coeficient of 0.68 was found. This implies that exponent n is about 1.9.
- In a regression of ln($A_{max}$) in terms of ln(v), a coefficient of 1.56 was found. This implies that exponent n is about 3.6.
- In a regression of ln(HIC) in terms of ln(v), a coefficient of 3.33 was found. This implies that exponent n is about 3.5. (Kessler and Monk, 1989, mentioned an empirical relationship in which HIC was proportional to $v^{2.93}$.)

### 7.8.3 Relationships that may apply to data for all materials

In addition, there are relationships that might apply to data for all materials, provided the materials differ only in stiffness k (and not in damping constant b or exponent n).

Firstly, HIC should be proportional to $A_{max}^{1.5}.v$.

- In a regression of ln(HIC) in terms of ln($A_{max}$) and ln(v), coefficients of 1.6 and 0.8 were found.
- A scatterplot of ln(HIC) versus ln($A_{max}^{1.5}.v$) shows an excellent straight line.
- The slope of that straight line is estimated to be 1.0.

(For HIC being proportional to $A_{max}^{1.5}.v$, see also sections 6.5.5, 9.3.3, 10.10.4, and 14.3.3.)

Secondly, HIC should be proportional to $S^{-1.5}.v^4$.

- Regression of ln(HIC) in terms of ln(S) and ln(v) found coefficients of -1.8 and 4.6.
- A scatterplot of ln(HIC) versus ln($S^{-1.5}.v^4$) shows an excellent straight line.
- The slope of that straight line is estimated to be 1.1.

Thirdly, $A_{max}$ should be proportional to $S^{-1}.v^2$.

- Regression of ln($A_{max}$) in terms of ln(S) and ln(v) found coefficients of -1.1 and 2.3.
- A scatterplot of ln($A_{max}$) versus ln($S^{-1}.v^2$) shows an excellent straight line.
- The slope of that straight line is estimated to be 1.2.

### 7.8.4 Discussion

This does not exhaust all the relationships that might be fitted to the data in Table 5 of Monk and Sullivan.

In addition, there is data in Table 6 that refers to impacts with 0.5 inch thick padding, and there is data in Table 7 that refers to impacts with 3 inch thick padding.

## *7.9 Fan (1983)*

Values of HIC from headform impact tests at two speeds (10 mile/h and 15 mile/h) with the interior of the A pillars of two cars were reported in Table 14 of Fan (1983). The slope of ln(HIC) versus ln(v) was about 2.3, corresponding to exponent n being about 0.8.

When the A pillar was padded with 1 inch of polyethylene foam, HIC was substantially reduced. The slope of ln(HIC) versus ln(v) was about 4.2, which is inconsistent with the proportionality relationship obtained in section 4.5. There may have been bottoming out of the padding at the higher speed but not the lower.

Some of the impacts in the dataset of Miller et al. (1996) (see section 7.6 above) were to the A pillar, and Miller et al. make some comments specifically about this. Also concerned with A pillars, there is some data from a study using the finite element method in Figure 12 of Naick and Carnago (1998).

## *7.10 DeMarco et al. (2010)*

### 7.10.1 Introduction

DeMarco et al. (2010) reported results of drop tests of ten designs of motorcycle helmets. Several aspects of this work are worth noting.

- Approved and non-approved designs of helmet were both included. (The non-approved helmets had no or little energy-absorbing material in the liners.)
- The impact speeds covered quite a wide range (1 m/sec to 10 m/sec).
- Coefficient of restitution was reported. Figure 4(d) of DeMarco et al. shows that it does not vary very much over a range from about 2 m/sec to 10 m/sec.
- Some of the helmet liners seemed to bottom out within the range of speeds tested.

DeMarco et al. showed interest in the functional form of the dependence of $A_{max}$ on impact speed. They fitted the following relationship.

$A_{max} \propto v$^(b + c.$v^d$)

Here, the symbol ^ means "raised to the power", so that x^n and x^(n) mean $x^n$. (That seems a reasonable solution to the typographical problem with exponents having exponents.)

### 7.10.2 Results

DeMarco et al. plotted $A_{max}$ versus v on linear axes. For two of the helmets (labelled by them as B3 and S3), the relationship appears curved, steepening considerably within the range of speeds tested. DeMarco et al. interpreted this as densification of the foam helmet liner (that is, bottoming out, or approaching it). DeMarco et al. describe B3 as a beanie helmet with an energy-absorbing liner 13 mm thick, and S3 as a shorty helmet with an energy-absorbing liner 25 mm thick.

I have plotted the data on logarithmic axes. Certainly there appears to be a steepening of the relationship at higher speeds, from a slope of about 1.2 to a slope of about 5.3. A slope of 1.2 implies exponent n (section 4.5) is 1.5. A slope of 5.3 is incompatible with the theory.

### 7.10.3 Discussion

Packaging engineers make use of a set of relationships summarised in a "cushion curve", that applies whether or not bottoming out is occurring. See sections 17.4.1 and 8.4.4 for this. It implies that ln($A_{max}$) is linearly dependent on $v^2$.

With this method of plotting, steepening of the relationship does not occur. For both the helmets (B3 and S3), the relationship is a good straight line.

## *7.11 Bonin et al. (2017)*

Bonin et al. (2017) conducted drop tests of three types of headform and four cadaver heads, wearing a single design of motorcycle helmet, at three speeds.

In the case of the Hybrid III headform, I have found the following results.

- Relationships between ln(v), ln($A_{max}$), and ln(HIC) were good straight lines. (That does not mean much with only three speeds, of course.)
- From the slope of the dependence of ln($A_{max}$) on ln(v), n was estimated to be 2.9.
- From the slope of the dependence of ln(HIC) on ln(v), n was estimated to be 1.9.
- The slope of the dependence of ln(HIC) on ln($A_{max}$) was estimated to be just less than 2. A true slope less than 2 would be inconsistent with the relationships of section 4.5. Thus it seems that n is very large.

## *7.12 Ghajari and Galvanetto (2010)*

Ghajari and Galvanetto (2010, Table 5) reported a comparison of finite element simulations of impacts of (a) a headform wearing a motorcycle helmet, and (b) a headform attached to a body and wearing a motorcycle helmet.

The results ($A_{max}$, HIC, S, maximum force) were slightly different. The question arises whether a change of mass of the headform could improve the agreement of the results with the results for the full body. Ghajari and Galvanetto showed the answer is yes, the agreement of all four output variables could be improved.

Ghajari and Galvanetto used some theory to obtain their conclusions. This included assuming (a) force is proportional to deformation, and (b) effective impact speed, which makes allowance for energy absorption by the helmet shell, is proportional to nominal impact speed.

The proportionality relationships of section 4.5.1 suggest alternative methods of estimating effective headform mass (along with exponent n) are practicable, without those assumptions being required. Since the need for an effective impact speed is plausible, a method that does not use impact speed would seem to be desirable.

## *7.13 Overview*

There are both positive and negative aspects to the data analysis above.

As has been seen in this chapter, extra insights into the data can be obtained by comparing it with theory. It is often practicable to ask and answer the following questions.

- Plotting the logarithms, is the scatterplot a straight line?
- What does the slope imply n is?
- When plotting the several dependent variables each versus the several independent variables, are the slopes compatible with a single value of n?
- If not, can the theory be rescued? When trying to improve the theory, is the approach of section 5.4 (some special phenomenon at initial contact) useful?
- In view of the theory, does any of the data look wrong?

Theory helps organise the data.

But on the other hand, it seems that theory can help only a little with present-day datasets.

- Measurement of the dependent variable is often less accurate than one might wish.
- There is often only one independent variable (e.g., impact speed).
- The independent variable often has only a limited range (i.e., the lowest value is quite a high fraction of the highest), and so the slope of the scatterplot is estimated only imprecisely. Consequently, the exponent n is likely to be estimated only imprecisely. (For further comment on estimating n, see Appendix 4.)
- There is often only one dependent variable (e.g., HIC).
- There is quite often reason to doubt the validity of one or more data points.
- Reporting of the experiment, both the methods employed and the results obtained, may omit some important details.

When one sees a scatterplot, and here I am thinking of untransformed variables for which zero is a special value, specific questions are likely to suggest themselves. (a) What is the shape of the relationship: in particular, is a straight line a good description, or does the relationship appear to be curved? To convincingly answer this, it is likely that a wide range of values of the independent variable will be needed. (b) What is the relationship when the independent variable is close to zero, and when the dependent variable is close to zero? This may be of interest either for its own sake, or for what it implies about the nature of the relationship (e.g., straight line or power function) across a wide range of values of the independent variable.

There are a number of other relevant studies not included in this chapter. But they do not dissuade me from the view that when confronted with the present theory, experimentation is clearly a limiting factor.

- In quantity primarily: not enough tests, not enough independent variables considered, not enough dependent variables considered, not a wide enough range of speed and other inputs.

- Also to some extent in quality: accuracy of measurement, repeatability, probable errors in reporting, insufficient detail in reporting.
- Data analysis includes: forming a reasoned opinion about which data points should be included and which excluded (as being unreliable or outside the scope of investigation); plotting the data points in ways that reveal qualitative and quantitative regularities; comparing these with theories and models; and thereby coming to an understanding of the dataset at hand. Dealing with data in that way seems hardly more than common sense. But a broad long-lasting culture of doing that does not seem to exist.

## *7.14 Comments on comparing data with theory*

Suppose that observations and theory agree.

- This is probably rather rare.
- It is consistent with the idea that the theory is correct and the observations mean what they are supposed to mean.
- But you can probably think of half a dozen good reasons why the observations might not mean what they are supposed to. If that is so, and these possible faults with the data have not been eliminated, it is questionable whether agreement of data with theory gives much support either to the data or to the theory.

Suppose the observations conflict with theory.

- This is probably much more common.
- However, I would not immediately regard the observations as discrediting the theory.
- The observations may be in error. Can they be made more accurately? Can some other method be used?
- The theory may be in error. Can it be extended or generalised so as to accommodate the observations? Can it be modified by mechanisms or phenomena that only occur in restricted circumstances? (For example, the beginning and the end of a series of observations may in some way differ from the others.) In referring to the theory being extended or generalised or modified, I am implying quite a minor change, that leaves the core of the theory intact.

Sometimes, numerical observations are reasonably accurate, and appropriate for purpose. But even in this near-ideal situation, comparison of theory and data may not be easy. Experimentation is often difficult, and though most observations may be correct, something may have gone wrong in a small proportion of cases. With some particular type of data, past experience might suggest (for example) that perhaps 5 per cent or 25 per cent of observations are likely to be wrong.

- There may be something wrong with an observation (an outlier, as it is often called) that appears inconsistent with the other observations. One should consider whether conclusions, or perhaps failure to come to any simple conclusions, are largely due to this observation.
- Outliers tend to draw attention to themselves, and so they receive careful attention. But there may be something wrong with an observation that appears consistent with the other observations. Again, it is desirable to consider whether conclusions are relying on this observation: if it were absent from the dataset, would the conclusions be unchanged? It is an unfortunate truism that when the number of observations is quite few, a very small number of erroneous observations can distort the overall message from the dataset.

My opinion is that it is very desirable to compare data with theory. But the benefits are not necessarily what you would expect if you are concentrating on confirming or disconfirming theory. Instead, theory helps organise, plan, interpret, and understand. There will be some further commentary in chapter 20.

# 8. Data from impact tests, 2: Sports (with or without helmets and other protection)

This chapter and the next are both about injuries that may occur in sports. Chapter 8 will examine some data relevant to sports helmets or crash mats, and chapter 9 some data relevant to impacts with the ground. Protection against a single impact is the concern, and multiple impacts are for the most part outside the scope of this book.

There are nine examples in this chapter.

## *8.1 Maw et al. (2012)*

### 8.1.1 Summary of experiments

Maw et al. (2012) were interested in protecting speed skaters wearing helmets who fall, slide, and strike a foam crash pad. They present a dataset giving maximum acceleration of Styrofoam impactor falling on to a crash pad at various speeds (actually, drop heights) and using impactors of various radii. What deforms is the foam of the crash pad, not the Styrofoam of the impactor.

### 8.1.2 Results

Theory applying to Hertzian impact has n = 1.5 and s = 0.5 (see sections 4.2.4 and 4.3.2).

On the basis of section 4.5, $A_{max}$ is expected to be proportional to $v^{2n/(n+1)}$. If n = 1.5, $A_{max}$ will be proportional to $v^{1.2}$.

Some results (Table 1 and Figure 1 of Maw et al.) may be summarised as follows.

- $A_{max}$ was about 20 g for a drop height of 1.0 m.
- Multiplying drop height by 13.3 (from 0.3 m to 4.0 m) multiplies maximum acceleration by 4.98 (this does not vary much with impactor radius). Multiplying drop height by 13.3 multiplies speed by approximately its square root, 3.65. $3.65^{1.24}$ is 4.98, so the exponent 2.n/(n + 1) is estimated to be 1.24.
- Multiplying impactor radius by 3 (from 10 cm to 30 cm) multiplies maximum acceleration by 1.20 (this does not vary much with drop height). $3^{0.17}$ is 1.2, so the exponent s/(n + 1) is estimated to be 0.17.
- Thus n = 1.62, and s is 0.17 × 2.62, which is 0.44.

In short, n = 1.62, s = 0.44 is a summary of the effects in Table 1 of Maw et al. As n is estimated to exceed 1, the crash pad may be described as a spring of increasing stiffness (increasing stiffness as it deforms more, that is). As s is positive, this supports the suggestion of Maw et al. (p. 820) that "larger and more blunt objects will recruit more foam more quickly to more rapidly decelerate", i.e., a greater radius of curvature will imply higher deceleration. This is made more precise by the empirical results being in quite close agreement with Hertzian theory, for which n = 1.5 and s = 0.5.

Expressing this in another way, if n = 1.5, the exponent 2.n/(n + 1) will be 1.2. If n = 1.5 and s = 0.5, the exponent s/(n + 1) will be 0.2. Exponents 1.24 and 0.17 are similar to the Hertzian values of 1.2 and 0.2.

### 8.1.3 Similar data

Table 2 and Figure 2 of Maw et al. present data for a different impactor, and d is what is called "cap radius". Nevertheless, the impacting surface is spherical, so it might be thought surprising that the relationships (between radius and maximum acceleration) are noticeably flatter. Reasoning similar to that of section 8.1.2 leads to n = 1.64, s = 0.15 as a summary of the effects in Table 2 and Figure 2. The flatter relationships are reflected in the smaller s.

Johnston et al. (2006) gave some results for impact speed, but not impactor radius. In their Figure 2(a), peak acceleration increases rather more sharply with speed than in Maw et al.: the two lines imply exponents of about 1.43 and 1.64, and hence n’s of about 2.5 and 4.6.

## *8.2 Lyn and Mills (2001)*

Lyn and Mills (2001) conducted impact tests of a headform with remoulded polyurethane foam. (This is used in crash mats for sports activities.) Drop heights were between 0.125 m and 1.0 m. The tests used three thicknesses of foam.

The following refers to thickness = 0.1 m, which was the thinnest tested. Deformations are somewhat larger (7 cm for a drop height of 0.5 m) and accelerations somewhat smaller (16 g for a drop height of 0.5 m) than in many experiments of this type.

- A plot of the dependence of ln($A_{max}$) on ln(h) is a good straight line, the slope being about 0.65.
- A plot of the dependence of ln(S) on ln(h) is a good straight line, the slope being about 0.30.
- A plot of the dependence of ln($A_{max}$) on ln(S) is a good straight line, the slope being about 2.1.

- The values of n implied by these slopes are respectively 1.8, 2.4, and 2.1. The estimates 1.8 and 2.4 are reasonably similar. (The third estimate is not independent of the other two, of course.)

On the basis of force versus deflection plots (on linear axes), the opinion of Lyn and Mills was that at the three greatest drop heights (for which deformation exceeded 60 per cent of the thickness of the foam), the foam was beginning to bottom out. The plot of $\ln(A_{max})$ versus $\ln(h)$ might be perceived as concave upwards, but it is only very slight.

The results of the tests of greater thicknesses of foam (0.2 m and 0.4 m) suggest rather smaller values of n (1.2 and 0.9).

## *8.3 Hrysomallis (2004)*

### 8.3.1 Summary of experiments

The co-variation of HIC and maximum acceleration is now considered for experiments in which d and v are constant.

Hrysomallis (2004) used a drop test from a height of 0.87 m, in this case of a headform wearing protective headgear on to a steel anvil covered with a rubber impact pad 1.3 cm thick. There were seven types of headgear and three orientations of impact. These combinations might be considered analogous to impacts at different locations; that is, the different results arise from k varying. The combinations are categories: headgear A, B, .... G, and orientations side, front, and top.

Hrysomallis gave both $A_{max}$ and HIC.

### 8.3.2 Results

In view of the theory given earlier (section 6.5.5), and assuming that only k varies, it may be asked whether HIC is proportional to $A_{max}^{1.5}$.

- Taking logarithms, there is an approximately straight-line relationship between ln(HIC) and $\ln(A_{max})$. The correlation is 0.86, even though the range of values of $A_{max}$ is quite narrow (from 176 to 249, a factor of 1.4). HIC and $A_{max}$ plainly vary together.
- The slope is 2.6, significantly different from 1.5.
- If separate regressions are carried out for the three orientations of impact, the slopes are 1.9, 1.6, and 2.7 (for side, front, and top). The first two are consistent with the theoretical value of 1.5.

In this case, then, the answer is ambiguous: it could be said that the slope is not 1.5, or alternatively that most of the data is compatible with a slope of 1.5, with impacts to the top of the headgear not consistent with this prediction. Possible variation in the coefficient of restitution has not been allowed for, and this may have obscured the relationship.

## *8.4 Gimbel and Hoshizaki (2008)*

### 8.4.1 Summary of experiments

Gimbel and Hoshizaki (2008) conducted drop tests on six materials used as the liner of children's helmets. They chiefly had in mind helmets for sports.

- Dependent variable: peak linear acceleration.
- Independent variables: 5 masses of impactors (headforms), at 5 speeds. Also, the 6 materials.

The resulting dataset has some coverage of both the "bottoming out" and the "no bottoming out" regions of material response to impact.

Gimbel and Hoshizaki were aware of the issues with bottoming out. However, they did not fit equations. Hutchinson (2014c) did so, using the results in their Tables I and II. More details of the results in sections 8.4.2 and 8.4.3 below are given in Hutchinson (2014c).

- Of the 150 possible combinations of conditions, Gimbel and Hoshizaki obtained results for 134.
- I thought it would be easier to understand the pattern of results if bottoming out impacts were distinguished from no bottoming out impacts.
- I classified 59 as no bottoming out (usually because, at the next higher impactor mass, peak acceleration was smaller), 36 as bottoming out (usually because, at the next lower impactor mass, peak acceleration was smaller), leaving 39 unclassified.
- A hypothesis of interest was whether peak force ($m.A_{max}$) was a power function of $m.v^2$. If that were so, $A_{max}$ would be proportional to $m^p.v^q$, with $q = 2.p + 2$.

### 8.4.2 Results: No bottoming out

I fitted a linear equation for $\ln(A_{max})$ in terms of ln(m), ln(v), and an intercept specific to each of the six materials. This is reasonable if the slopes are the same for the six materials: in effect, the estimates of the slopes are averaged over the six materials.

- The two slopes were estimated as -0.73 and 0.50.
- The exponent of v is indeed approximately $2 \times (1 + \text{exponent of m})$.
- The slopes would be -0.5 and 1 for the linear spring ($n = 1$), so this hypothesis is not supported. Instead, force during the impact is approximately proportional to distance of deformation raised to the power 0.33.

### 8.4.3 Results: Bottoming out

Again, I fitted a linear equation for ln($A_{max}$) in terms of ln(m), ln(v), and an intercept specific to each of the six materials.

- The two slopes were estimated as 1.84 and 2.76.
- The exponent of v is not approximately 2 × (1 + exponent of m).

### 8.4.4 Results: A single relationship

Packaging engineers make use of a set of relationships summarised in a "cushion curve", that applies whether or not bottoming out is occurring. See section 17.4.1 for this, and also 7.10.3. It implies that ln(m.$A_{max}$) is linearly dependent on m.$v^2$.

When plotting ln(m.$A_{max}$) versus m.$v^2$ for each of the six materials (separately), the relationship was a good straight line. Beyond that, different people are likely to hold different opinions.

- Some people will say there is no reason to consider any more complex relationship.
- Others will say they can perceive failure of linearity at both ends of the scatterplot. They are likely to agree there is a straight-line relationship for the middle of the range of m.$v^2$. But they will say that for very low m.$v^2$, the data points are below that straight line. And for high m.$v^2$, either the data points are generally more scattered, or there are a few outliers above the straight line.

My impression is that unfortunately it is rather rare for bottoming out to be studied, and even more rare for the cushion curve or similar method to be employed, other than in the context of packaging. Examples are Gilchrist and Mills (1994) and Shuaeib et al. (2002).

## *8.5 Stalnaker and Rojanavanich (1990), and Stone et al. (2016)*

Stalnaker and Rojanavanich (1990) used an air cannon to fire softballs (which are hard) at a Hybrid III head mounted on a neck that was mounted rigidly. (The Hybrid III is a well-known crash test dummy.) Headform accelerations were measured and HIC calculated. Thirteen softballs of different construction (core and cover) were used, each at 3 speeds of impact. This part of the study did not use helmets. Three speeds are very few for the purpose of judging whether a relationship is a straight line or not. Nevertheless, there were 13 sets of 3. Consequently, I feel able to say that the relationships between ln(v) and ln(HIC) did appear to be straight lines. The slope was about 2.6, which corresponds to exponent n being about 1.2.

Stone et al. (2016) used an air cannon to fire a cricket ball at a headform (wearing a helmet) that was either on a fixed mounting or was free to move. There were three impact positions. For the six experimental conditions, average values of $A_{max}$ were given at three speeds. The slope of $\ln(A_{max})$ vs. $\ln(v)$ may not be the same for the six experimental conditions. If, nevertheless, a single slope is fitted, it is about 1.4, corresponding to exponent n being about 2 or 3.

## *8.6 Ouckama and Pearsall (2014)*

### 8.6.1 Summary of experiments

Ouckama and Pearsall (2014) fired an ice hockey puck at the front of helmets worn by an instrumented headform.

They used five models of helmet and two impact speeds (24.2 m/sec and 33.0 m/sec) in their experiments. They reported several proxies for injury, including $A_{max}$. (They seem to regard strain and rotational measures as preferable to $A_{max}$ and other translational measures.)

### 8.6.2 Results

The ratio of impact speeds was 1.36. The ratio of $A_{max}$ at the two speeds varied quite considerably between the five models of helmet, from 1.61 to 2.47.

- A ratio of 1.61 corresponds to exponent n being about 3.2.
- A ratio of 2.47 is incompatible with the theory of chapter 4 based on the Hunt and Crossley equation.

Another of the proxies for injury reported was maximum pressure. (This was measured with an array of 25 force sensors; see also Ouckama and Pearsall, 2012.) The ratio of the maximum pressures at the two speeds varied quite considerably between the five models of helmet, from 1.77 to 3.93. I presume that maximum pressure will behave as maximum acceleration does, in the sense that it will be proportional to $v^{2n/(n+1)}$.

- A ratio of 1.77 corresponds to exponent n being about 12.
- A ratio of 3.93 is incompatible with the theory of chapter 4.

### 8.6.3 Discussion

Not surprisingly, in view of the range of ratios, there is cross over of dependence of proxy for injury on impact speed. For example, helmet model 4 was the best at the lower impact speed in the sense that maximum pressure was lower than for the other helmets, but it was the worst at the higher impact speed in the sense that maximum pressure was higher than for the other helmets.

Bottoming out of helmet model 4 is a possible explanation, or the laws governing impact behaviour may be different for the several bonnets. See section 2.3 for some text relevant to bottoming out and cross over of functions representing dependence of injury severity on impact speed.

## *8.7 Pearce and Young (2014)*

### 8.7.1 Summary of experiments

Pearce and Young (2014) used finite element models to study head impacts of duration less than about 1 msec. These may occur when a projectile of low mass strikes the head. Pearce and Young probably had in mind balls used in sports, or perhaps less-lethal projectile weapons. There was a series of impacts having the same kinetic energy, impactor velocity being between 0.2 m/sec and 7.0 m/sec, and impactor mass being between 8.0 kg and 0.0065 kg.

Pearce and Young were chiefly interested in intracranial pressure. They also reported pulse duration and maximum force.

For all impactors except one, mass was less than 1 kg, and thus considerably less than head mass.

### 8.7.2 Results: Maximum force

The proportionality relationships in section 4.5.1 suggest that maximum force may be proportional to $m.A_{max}$, that is, to $m.m^{-1/(n+1)}.v^{2n/(n+1)}$ $= m^{n/(n+1)}.v^{2n/(n+1)}$. As this is a function of kinetic energy, $½.m.v^2$, and this was kept constant in the experiments, maximum force may be constant.

To a large extent, that was the case. For the 10 impactors having mass between 0.0065 kg and 0.32 kg, maximum force was between 0.96 and 0.98 kN. For the impactors of mass 0.9 kg and 8.0 kg, maximum force was 0.87 kNand 0.47 kN, respectively. (I presume the head was free to move, and thus the velocity change of impactors of high mass would be less than that of impactors of low mass.)

### 8.7.3 Results: Pulse duration

The proportionality relationships in section 4.5.1 suggest that pulse duration may be proportional to $m.A_{max}$, that is, to $m^{1/(n+1)}.v^{-(n-1)/(n+1)}$, which may be rewritten as $(m.v^2)^{1/(n+1)}.v^{-1}$. Thus as $½.m.v^2$ was kept constant, pulse duration may be inversely proportional to impact speed v in these experiments. That is, the product of duration and v may be constant.

To a large extent, that was the case. For the 10 impactors having mass between 0.0065 kg and 0.32 kg, the product v.T was between 1.18 mm and 1.08 mm. For the impactors of mass 0.9 kg and 8.0 kg, the product v.T was 1.01 mm and 0.66 mm, respectively.

## *8.8 Ahmad et al. (2009)*

Ahmad et al. (2009) reported on characteristics of the movement of a sepak takraw ball (observed on high-speed video) during impact with a force plate. This analysis is not directly concerned with any injury that may occur (if the ball hits someone's head, for example), as what is being accelerated and deformed is the ball, not a proxy for a person. Mechanical properties of the ball that may be discovered or measured are quite likely to be relevant to potential injury, however.

The results show that when impact speed increased by a factor 1.8, maximum deformation of the ball increased by a factor 1.69, and maximum force increased by a factor 1.66. These results respectively imply that exponent n is 1.25, and that n is 0.75. Those values seem not very similar, but n = 1 would imply both of the relevant exponents would be 1 also, and so maximum deformation and maximum force would both increase by a factor 1.8, which is not very different from 1.69 and 1.66.

A Japanese-language paper by Arakawa et al. (2006) studies the deformation of a golf ball (observed on high-speed video) striking a rigid steel target. Arakawa et al. were aware that Hertzian impact implies that deformation is proportional to $v^{0.8}$ and maximum force is proportional to $v^{1.2}$. Their Figures 5 and 12 show good empirical agreement with those hypotheses.

# 9. Data from impact tests, 3: Sports (ground impact)

There are six examples in this chapter.

Some surfaces on to which a person may fall (perhaps from approximately standing height) are concrete or other rigid material. For relevant experiments, see chapter 14.

## *9.1 Viano et al. (2012)*

### 9.1.1 Summary of experiments

Viano et al. (2012) studied impact attenuation by natural and artificial turf surfaces (or, rather, attenuation by the combination of turf and helmet). Their experiments involved dropping an instrumented headform, wearing an American football helmet, from two heights on to various surfaces. They used both maximum acceleration and HIC (the Head Injury Criterion) as indicators of potential injury.

The exponents reported below were calculated from the data in Table 3 of Viano et al. Tests were at two speeds (drop heights) on to several types of natural or artificial turf. Results (averages of six tests) were given for 34 series of tests. I will use "set" to refer to a pair of series of tests, at 4.2 m/s and at 6.0 m/s. There were 17 sets of tests. Some sets were in mild conditions, others in cold conditions.

Three relationships will be examined: between speed v and $A_{max}$, between v and HIC, and between $A_{max}$ and HIC. These all ought to imply the same value of n. This prediction will be found to fail, and in section 9.1.5 a modification of the theory will be suggested.

Firstly, each set or pair of results will be considered, and the effects of changing speed summarised. Secondly, the co-variation of $A_{max}$ and HIC across sets at each speed will be summarised.

### 9.1.2 Results: Change of speed

In this section, consideration is given to the dependence of $A_{max}$ and of HIC on v, and to the relation between HIC and $A_{max}$ as v changes, for each set.

Only two speeds were used, so it is not possible to check whether the relationships are power functions (straight lines after taking logarithms).

It can be asked, however, whether the slopes for logarithmic plots are within the possible ranges. Predictions are as follows.

- The ratio of change in ln($A_{max}$) to change in ln(v) is 2.n/(n + 1), and so (as n is positive) should be between 0 and 2. If (as examples) n is 0.5, 1, or 1.5, this exponent would be 0.67, 1, or 1.2.
- The ratio of change in ln(HIC) to change in ln(v) is (4.n + 1)/(n + 1), and so should be between 1 and 4. If n is 0.5, 1, or 1.5, this exponent would be 2, 2.5, or 2.8.
- The ratio of change in ln(HIC) to change in ln($A_{max}$) is (4.n + 1)/(2.n), and so should be at least 2. If n is 0.5, 1, or 1.5, this exponent would be 3, 2.5, or 2.33.

The results agreed with these predictions, for all sets except for that corresponding to test series 5 and 6. The results from changing impact speed are thus consistent with the theory for 16 of the 17 sets. (The exception was an Astroplay field that was the best performer at the lower speed of impact, having the lowest $A_{max}$ and the lowest HIC, and was good but not the best at the higher speed of impact.)

The following additional results refer to the 16 sets.

- The ratio of change in ln($A_{max}$) to change in ln(v) had a median of 1.13. Let nAv be the n calculated from this ratio; nAv was found to have a median of 1.31, and to be greater than 1 for 14 of the 16 cases.
- The ratio of change in ln(HIC) to change in ln(v) had a median of 2.92. Let nHv be the n calculated from this ratio; nHv was found to have a median of 1.78, and to be greater than 1 for 15 of the 16 cases.
- The ratio of change in ln(HIC) to change in ln($A_{max}$) had a median of 2.62. Let nHA be the n calculated from this ratio; nHA was found to have a median of 0.80, and to be greater than 1 for 5 of the 16 cases.
- There were systematic differences between the values of n implied by the three pairs of changes: nHv tended to be greatest, and nHA tended to be the smallest. It is difficult to know how seriously to take the difference of nHA from nAv and nHv. As mentioned in sections 6.5.1 and 6.7.3, random error can have a big effect on nHA: see Appendix 5.
- The three estimates of n were correlated as follows: the correlation of nAv and nHv was 0.92, that of nAv and nHA was 0.77, and that of nHv and nHA was 0.48.

### 9.1.3 Results: Change of surface

In this section, consideration is given to how $A_{max}$ and HIC co-vary across sets, at each speed. (Thus the variation was between different types of turf, different stadiums, different locations on the field, and different temperatures.) The prediction in this case is that HIC is proportional to deltaV.$A_{max}^{1.5}$ (where deltaV refers to the change in speed). It is greater than v because of bounce. That is, coefficient of restitution is not 0; it averaged 0.58 at the lower speed and 0.53 at the higher speed.) At each impact speed separately, there is little variation in deltaV across sets, and consequently HIC should be approximately proportional to $A_{max}^{1.5}$.

Regressions of ln(HIC) vs. ln($A_{max}$) were carried out. Results were very similar whether HIC was used or the ratio HIC/deltaV, and whether test series 5 and 6 were excluded or included.

For the lower impact speed, the dataset was consistent with the prediction (estimated exponent = 1.48). For the higher impact speed, the exponent was a little greater than 1.5 (estimated exponent = 1.72, with a standard error of 0.07).

### 9.1.4 Interpretation

The theory given in chapter 4 is partially successful. There was a high correlation between the exponents calculated from the effects of v on $A_{max}$ and HIC. Further, across sets, $A_{max}$ and HIC co-vary in the way expected at the lower speed.

But on the other hand, there were systematic differences between the values of n calculated from the three pairs of changes. The coefficient of restitution tended to be a little smaller (by 0.05 on average) at the higher speed than at the lower speed, which is contrary to the theory. Across sets, the dependence of ln(HIC) on ln($A_{max}$) was steeper than expected at the higher speed.

### 9.1.5 Modification of theory

The theory will now be modified, to try to ensure that within sets, the values of n implied by the three pairs of changes are consistent with each other. The following suggestions are made.

- The value calculated from the ratio of change in ln(HIC) to change in ln($A_{max}$) (that is, nHA) is the most important, as it is calculated from high values of acceleration and is unaffected by low accelerations.
- Some materials are softer close to their surface than through their bulk (perhaps because particles are only loosely packed at the surface). Some energy is absorbed at the top of the turf, before high values of acceleration are reached. To take account of this, there should be a concept of effective impact speed as in section 5.4.2, calculated as $\sqrt{(v^2 - v_c^2)}$, where $v_c$ is likely to be different for different materials. This formula assumes the same amount of energy is absorbed whatever v is.

- As mentioned in section 6.7.3, the ratio of the higher to lower impact speed is less than the ratio of higher to lower effective impact speed. Thus the ratio of change in ln(A) to change in ln(v) is larger than it should be, and nAv is larger than it should be. The same is true for nHv.

The exponent nHA will not be affected by v being replaced by effective impact speed. Thus nHA can be treated as known from the analysis in section 9.1.2: there was some variation between surfaces, but n was about 0.80. Then for 15 of the sets a critical velocity $v_c$ was defined such that n calculated from the ratio of change in $\ln(A_{max})$ to change in $\ln(\sqrt{(v^2 - v_c^2)})$ equals nHA; n calculated from the ratio of change in ln(HIC) to change in $\ln(\sqrt{(v^2 - v_c^2)})$ then equals nHA also. The 15 quantities $v_c^2$ ranged from 2.0 to 6.6; thus $v_c$ ranged from 1.4 to 2.6 m/sec. For each surface, $v_c$ is the speed such that below this there is effectively no impact. These speeds are quite small, and seem reasonable.

Two sets have been omitted. For that corresponding to test series 5 and 6, the ratio of change in ln(HIC) to change in $\ln(A_{max})$ was less than 2. For that corresponding to test series 3 and 4, nHA was greater than nAv.

On the new interpretation of the results of Viano et al., some energy of the headform is absorbed close to the top of the turf, and $A_{max}$ and HIC are power functions of effective impact speed that are connected through having n in common. The exponent n is about 0.8 and the critical speed $v_c$ is about 2.0 m/sec, though it varies from one material to another. Although, on this interpretation, there is a change from low accelerations to high, this would not fall within the usual meaning of bottoming out: the change, or distortion of results, occurs at low speeds, the peak accelerations are too low (less than 170 g), and the exponent n was less than 1 in most cases.

There is no guarantee that this interpretation is the correct one.

- It might instead be claimed that results at the higher speed are affected by bottoming out, that the high values of nAv and nHv should be taken at face value, and that the differences between the values of n calculated from the three pairs of changes should be taken as a warning that the theory proposed is not valid; perhaps, in view of the complex nature of bottoming out, it is unlikely that any reasonably simple theory will be satisfactory. This view would suggest that what is needed is to understand better the high values of $A_{max}$ and HIC, not to change the understanding of low values by introducing $v_c$.
- Another possibility is that nHA is substantially in error (see Appendix 5).

At present, a reasonably simple proposal has been made based on $v_c$ and n. Better evidence for this and against alternative interpretations would require more data (e.g., at more speeds, or concerning distance S and time T). This should not be taken as implied criticism of the dataset of Viano et al., which is very valuable for having 17 sets at two speeds, and giving $A_{max}$, HIC, and deltaV.

## *9.2 Murayama et al. (2013)*

### 9.2.1 Summary of experiments

Murayama et al. (2013) used a method in which a whole dummy (with an accelerometer in the head), rather than a headform, was thrown by a judo expert on to a tatami (judo mat), either without or with a polymer under-mat beneath it, laid on a concrete floor. The data to be considered here was given on p. 582 of their paper.

Murayama et al. defined $A_{max}$ as the maximum acceleration that was maintained for at least 3 msec. The derivation of the proportionality relationship in section 4.5 will not quite apply in this case, as 3 msec will not be a constant fraction of the pulse duration. The inaccuracy from this source will be small, however, as there is typically very little variation in pulse duration. Impact speeds were not reported. What is analysed below is the co-variation of $A_{max}$ and HIC.

### 9.2.2 Effect of an under-mat

In this section, for a particular throwing technique, the effect of the under-mat is considered. When the Osoto-gari technique was used, the effect of the under-mat was to reduce $A_{max}$ by 38 per cent, and reduce HIC by 53 per cent, implying an exponent of 1.58.

That conclusion was presented very briefly. I will spell out the mathematical manipulations.

- Firstly, I will reword the empirical finding. When the Osoto-gari technique was used, the effect of the under-mat was to reduce $A_{max}$ to 0.62 of what it was, and to reduce HIC to 0.47 of what it was.
- Secondly, if HIC $\propto A_{max}^c$, the ratio of the two values of HIC will be the ratio of the two values of $A_{max}$ raised to the power c. That is, $0.47 = 0.62^c$.
- Therefore, $c = \ln(0.47)/\ln(0.62)$, which is 1.58.

With the Ouchi-gari technique, the effect of the under-mat was to reduce $A_{max}$ by 38 per cent, and reduce HIC by 54 per cent, implying an exponent of 1.63.

Both exponents are close to 1.5, which is what would be expected if the effect of the under-mat were to change k. (See section 6.5.5.)

### 9.2.3 Effect of throwing technique

Now, for each under-mat condition (without or with a mat), consider what effect the throwing technique has.

Without an under-mat, a change from Ouchi-gari to Osoto-gari reduced $A_{max}$ by 51 per cent, and reduced HIC by 72 per cent, implying an exponent of 1.78.

With the under-mat, a change from Ouchi-gari to Osoto-gari reduced $A_{max}$ by 51 per cent, and reduced HIC by 71 per cent, implying an exponent of 1.75.

These exponents are less than 2, which is the minimum that would be expected if the effect of throwing technique were to change the impact velocity. The exponents are also greater than 1.5, which is what would be expected if the effect of throwing technique were to change the effective mass of the head at impact.

According to Murayama et al. (p. 583), head impact speed is higher with Ouchi-gari than with Osoto-gari. If higher speed is accompanied by lower effective mass of the head, an exponent between 1.5 and 2 could occur.

### 9.2.4 A dummy or a headform?

My opinion is that it would be helpful if expert experimenters would spell out their reasons for choosing a dummy or a headform. I do not know, but I would expect the respective arguments to be roughly as follows.

- What is being accelerated in the real-life impact is the whole person. Therefore, a dummy should be used in a test or experiment.
- The most severe injuries in real-life impacts are head injuries. These usually occur when the impact is largely concentrated on the head. In such circumstances, the effective head mass is only a little greater than actual head mass. Therefore, a headform should be used.

Which of these is most realistic varies from context to context. It is very likely, of course, that other reasons (such as realism, repeatability, and practicability) also contribute to the choice between dummy and headform.

The choice will matter a great deal if much of the dummy's mass is relevant to the acceleration being measured. A dummy's mass might be (say) 16 times a headform's mass. If exponent n is 1, maximum deformation is proportional to $m^{0.5}$, and maximum acceleration is proportional to $m^{-0.5}$ (see section 4.5.1). Thus multiplying m by 16 will increase S by a factor of 4 and will decrease $A_{max}$ by a factor of 4.

As noted in section 4.6.1, greater mass of impactor means lower maximum acceleration, with the reservation that it also means higher deformation, and thus a greater likelihood of sudden bottoming out and very high maximum acceleration.

Petrone (2012), like Murayama et al., used a dummy rather than a headform. Petrone conducted experiments on safety barriers intended to protect skiers. The wearable airbag experiments of Fukaya and Uchida (2008), to be described in section 11.1, also employed a whole dummy.

## *9.3 Shields and Smith (2009)*

### 9.3.1 Summary of experiments

Shields and Smith (2009, Table 2) reported $A_{max}$ and HIC for impacts of an instrumented headform on a number of surfaces that cheerleaders might fall on to.

- For each of twelve surfaces, data for four impacts (i.e., four sites on the surface) at the same drop height (i.e., the same speed) are given.
- The surfaces differed in stiffness, and the drop heights were different for the different surfaces. For ten of the twelve surfaces, $A_{max}$ was over 200 g and HIC was approximately 1000, and for the two softest surfaces $A_{max}$ and HIC were rather less than those values.

Are there any regularities in the data for the four impacts on a surface? There might not be: the purpose of four drops rather than one was to assess the variation for that surfacing material, and variation might be random and not explainable. It turns out, though, that there is correlation, sites for which $A_{max}$ was high tending to have high HIC also.

Strictly speaking, I should say that acceleration pulses for which $A_{max}$ was high tended to have high HIC also. This does not imply that site-to-site variation is responsible. Something else about the impact may have been the cause: possibly small variations in impact speed, or orientation of the headform, or behaviour of the accelerometer, or something else. But Shields and Smith intended the four replications to reflect the variation in the surfacing material, and there is no reason to doubt this.

For a given surface, all four sites have the same impact speed. It may be reasonable to assume velocity change is constant, too. (On some surfaces there may be no bounce back at all, and velocity change will equal impact speed.) Then HIC will be proportional to $A_{max}^{1.5}$. (See also section 6.5.5.)

### 9.3.2 Results, each of the surfaces

For each of the twelve surfaces having data for four sites (Table 2 of Shields and Smith, 2009), straight line dependence of ln(HIC) on ln($A_{max}$) was fitted.

- In the case of the four very stiff surfaces for which the drop height was 0.5 or 1.0 ft, the slope of the line was between 1.50 and 3.43.
- In the case of the eight less stiff surfaces for which the drop height was at least 1.5 ft, the slope of the line was between 1.21 and 1.65, and averaged 1.45.
- As expected, the individual slopes are estimated quite imprecisely: the standard errors averaged 0.18.

The following interpretation is suggested.

- For many of the surfaces, the main source of variation is site-to-site variation in the surface, there is usually little variation in bounce, and consequently HIC is approximately proportional to $A_{max}^{1.5}$.
- In the case of very stiff surfaces and small drop heights, possible reasons for failure of the theory might include the impact-to-impact error in drop height (and thus in impact speed) being proportionately larger, or the site-to-site variation in rebound speed being greater for stiff surfaces.

The dataset of Shields and Smith is in some ways unusual: there is no systematic change of some independent variable (e.g., drop height) with the intention of determining its effect on a dependent variable (e.g., $A_{max}$). Thus there may have been no interesting regularities in the data. But the dataset has strengths, including the four repetitions of impact in the same conditions (same drop height and same surface), and the use of surfaces of a wide range of stiffnesses. And it turned out that the data did show correlations between $A_{max}$ and HIC that presumably reflect minor site-to-site variation in the surfacing material and are quantitatively consistent with that for the majority of surfaces. (Section 9.3.1 mentioned some reasons why spurious correlation between $A_{max}$ and HIC might possibly occur, and these possibilities have not been eliminated.)

### 9.3.3 Results, all surfaces considered together

HIC is predicted to be proportional to $A_{max}^{1.5}$.v (see sections 6.5.5, 7.8.3, 10.10.4, and 14.3.3). As v is proportional to $\sqrt{h}$ (h = drop height), HIC/$\sqrt{h}$ should be proportional to $A_{max}^{1.5}$. Thus when ln(HIC/$\sqrt{h}$) is plotted versus ln($A_{max}$), the slope should be 1.5.

When ln(HIC/$\sqrt{h}$) was plotted versus ln($A_{max}$), with all the surfaces included, a good straight line resulted. The four data points for the surface described as "Landing mat on vinyl tile" were exceptions. For these, $\ln(HIC/\sqrt{h})$ was appreciably smaller than for other impacts having similar values of $\ln(A_{max})$. I have no other reason to think drop height was recorded incorrectly in Table 2 of Shields and Smith for this surface, but these data points would fall in line with the others if h was about 2.0 ft instead of the stated 6.5 ft.

With those four data points excluded, the slope of the remaining points was 1.62, which is not far from the predicted value of 1.5.

## *9.4 Theobald et al. (2010)*

### 9.4.1 Summary of experiments

Theobald et al. (2010) reported measurements of HIC obtained by dropping a headform from various heights h on to artificial and natural turf surfaces. They used a 4.5 kg headform in their tests.

Straight lines through the origin are shown on Figures 3 and 4 (HIC plotted against h) of Theobald et al. In terms of the theory of chapter 4, these lines of proportionality show the fit of the theory with (4.n + 1)/(2.n + 2) being assumed to be 1, that is, n = 0.5.

### 9.4.2 Results

The visual impression given by many of the plots (Figures 3 and 4 of Theobald et al.) is that they are concave upwards, suggestive of a power function with exponent greater than 1. This would imply that n is greater than 0.5.

That may be confirmed as follows.

- In Figure 4 of Theobald et al., HIC is plotted versus h. This was from in situ tests of one artificial turf. Taking logarithms and plotting ln(HIC) versus ln(h), the plot may be seen to be close to a straight line. The slope is 1.78, implying that HIC is approximately proportional to $h^{1.78}$.
- In Figure 3 of Theobald et al., there are 12 plots of HIC versus h. These were from laboratory tests of six artificial turfs, each in fresh and bedded-in conditions. As a quick summary of how HIC increases in each plot, HIC at h = 0.8 m may be compared with HIC at h = 1.6 m. In Figure 3 of Theobald et al., HIC can be seen to be typically multiplied by 3 as a consequence of this doubling of h. This implies that HIC is approximately proportional to $h^{1.59}$ (as $2^{1.59} = 3$).
- The first result implies n is about 5.8, and the second result implies that n is about 2.7.

There is data of a similar form in Figure 1 of Dickson et al. (2018). For three surfaces, when h increased by a factor of 5, HIC increased approximately linearly by factors of about 10.5, 8.9, and 8.6.

### 9.4.3 Alternative interpretation

As in section 9.1.5, it is worth considering the alternative hypothesis that the turf has a soft top to the surface that absorbs a little energy, with most deceleration taking place later, in the body of the turf.

As suggested in section 5.4.2, this might be modelled as an effective impact speed of $\sqrt{(v^2 - v_c^2)}$ (instead of v), where $v_c^2$ reflects the energy absorbed in the top of the surface, does not depend on v, and may depend on which turf is being considered.

- HIC will be proportional to $(v^2 - v_c^2)^{(4n+1)/(2n+2)}$.
- In principle, three parameters --- the constant of proportionality, the exponent n, and $v_c$ --- could then be fitted to the plots of HIC versus h (since $v^2$ is proportional to h). In practice, however, three are probably too many, in the sense that quite different sets of parameters may give approximately equally good fits.
- It might be reasonable to assume that n = 1.5, as for Hertzian impact.
- If n = 1.5, HIC is proportional to $(v^2 - v_c^2)^{7/5}$. Consequently, $HIC^{5/7}$ has linear dependence on $v^2$ (and on h), and $v_c$ can be estimated.

Whether ln(HIC) is plotted versus ln(h) or $HIC^{5/7}$ is plotted versus h, about equally good straight lines are obtained. That is, to understand the results of Theobald et al., it might be assumed that $v_c = 0$ with n fitted to the data, or that n = 1.5 with $v_c$ fitted to the data. The dataset of Theobald et al. is not quite extensive enough to discriminate between these possibilities.

## *9.5 Martin et al. (1994)*

Martin et al. (1994) reported drop tests on to dry sod, moist sod, and an artificial playing surface. The surface of the impactor was hemispherical. Force was measured with a force plate under the surface, which was 2.5 cm to 4.5 cm thick; acceleration was calculated from force. It is plain that Martin et al. regarded their method as validly measuring acceleration. A single drop height was used, but two masses of impactor.

Exponent n may be calculated from the change of $A_{max}$ relative to the change in mass (Table 1 of Martin et al.). For the three surfaces, it was 0.9, 5.6, and 2.5.

## *9.6 Pain et al. (2005)*

Pain et al. (2005) reported the results of drop tests of a 24 kg impactor on to a gymnastics landing mat. There were five impact speeds, the highest being about 50 per cent greater than the smallest.

The following uses data from Tables 1 and 3 of Pain et al. The plot of ln(S) vs. ln(v) is a good straight line, and the slope is about 0.7. Using $A_{max}$ measured by an accelerometer, the plot of ln($A_{max}$) vs. ln(v) is a good straight line, and the slope is about 1.0. These slopes respectively suggest exponent n is about 1.8 and 1.0, apparently somewhat different, but n = 1.4 is compatible with both slopes.

Much of the paper by Pain et al. is concerned with comparing results from an accelerometer, a force plate, and high-speed video.

Comparison of acceleration (from the accelerometer or video) with force (from the force plate) permitted the calculation of an effective mass of the mat.

# 10. Data from impact tests, 4: Children's play (ground impact)

## *10.1 Introduction*

The examples in this chapter are of the testing of surfaces that children may fall on to while playing.

Some surfaces on to which a person may fall (perhaps from approximately standing height) are concrete or other rigid material. For relevant experiments, see chapter 14. The example in section 14.7 concerns low-height falls of infants.

It may be noted that at present child headforms have a mass of typically 2.5 or 3.5 kg. The headforms used in the experiments discussed below were all rather heavier than this. As noted in section 4.6.1, headform mass has two contrasting effects. Provided sudden bottoming out does not happen, greater mass means lower maximum acceleration and HIC. (See section 4.5 for these relationships.) However, greater mass also means higher deformation, and thus a greater likelihood of sudden bottoming out.

According to Chang and Huang (2007, p. 848) the choice of a 4.6 kg headform in some Standards is deliberate. In the context of helmets, the opinion of Gimbel and Hoshizaki (2008) was that Standards ought to specify a headform mass that reflects the head mass of the intended user group. (Data in Gimbel and Hoshizaki, relevant particularly to children's helmets, was discussed in section 8.4.)

As mentioned earlier (section 6.3.2), drop height h is often used instead of impact velocity. Impact velocity is assumed to be proportional to the square root of drop height. Thus a hypothesis that HIC (for example) is proportional to $v^{(4n+1)/(n+1)}$ becomes a hypothesis that HIC is proportional to $h^{(4n+1)/(2n+2)}$. With n being positive, the exponent of h is between 0.5 and 2.

There are nine examples in this chapter.

## *10.2 Vidair et al. (2007a, b)*

### 10.2.1 Summary of experiments

Vidair et al. (2007a, b) reported drop tests of a headform at playgrounds having rubberised surfaces. Four subsets of data give dependences of HIC on fall height: two playgrounds, each at two temperatures. These are shown in Figures 13 and 14 of Vidair et al.

(2007a), and Figure 4 of Vidair et al. (2007b). Vidair et al. used a 4.5 kg headform.

Vidair et al. do not report the values of $A_{max}$ along with those of HIC.

### 10.2.2 Results

A plot of ln(HIC) vs. ln(h) is a good straight line in each of the four cases. Thus HIC is approximately a power function of drop height. The exponent was estimated as 1.94, 2.04, 1.44, and 1.46 in the four cases.

For the first playground, the exponents of 1.94 and 2.04 (at the two temperatures) are compatible with n being about 10, though they might be considered so high as to cast doubt on the appropriateness of the theory.

For the second playground, the exponents of 1.44 and 1.46 (at the two temperatures) imply that n is 1.68 and 1.78 in these two cases.

### 10.2.3 Alternative interpretation

Sections 9.1.5 and 9.4.3 used a concept of effective impact speed. This was needed because of energy hypothesised to be absorbed in the top of the surface, and not responsible for the peak acceleration or for HIC.

For the present dataset, $A_{max}$ is not available and hence n cannot be calculated from HIC and $A_{max}$ (without use of v), as in section 9.1.5. That being so, it might be assumed that n is 1.5, as for Hertzian impact (see sections 4.2.4 and 9.4.3). This would imply that $HIC^{5/7}$ plotted against h would be a straight line.

Whether ln(HIC) is plotted versus ln(h) or $HIC^{5/7}$ is plotted versus h, about equally good straight lines are obtained. (As expected, when $HIC^{5/7}$ was plotted against h, the intercept on the horizontal axis was positive.) That is, either an effective speed is not used and n is fitted to the data, or n is assumed to be 1.5 with a quantity $v_c$ (as in section 9.4.3) fitted to the data. Thus (as with the dataset of Theobald et al. in section 9.4) the dataset of Vidair et al. is not extensive enough to discriminate between these possibilities.

### 10.2.4 Possible bottoming out

Vidair et al. show concern with surface thickness, including a remark that "failures may have resulted from installation of surfaces that were not sufficiently thick", and (for the minority of surfaces that were of loose-fill shredded tyre rubber) comment that some high HIC values refer to locations where most of the rubber had been kicked away. (See pp. 109-113 of Vidair et al., 2007a, and p. 229 of Vidair et al., 2007b.) Thus bottoming out may have been a problem with the surfaces.

## *10.3 Ghani and Rased (2014)*

Ghani and Rased (2014) reported laboratory experiments in which a headform of mass 4.6 kg was dropped on to tiles manufactured from one of several formulations of rubber. There were six types differing in respect of the sizes of the pieces of waste rubber used, and the percentage of styrene butadiene rubber added. In some impacts there was a plywood cover to the rubber. Four drop heights were used, from 0.3 m to 1.0 m, and $A_{max}$ and HIC were both reported.

A plot of ln($A_{max}$) versus ln(drop height) is a good straight line. On fitting straight lines with the same slope but different intercepts, the six formulations of rubber did not differ in respect of intercept. The slope was estimated as 0.64, implying an exponent n of approximately 1.7.

A plot of ln(HIC) versus ln(drop height) is a good straight line. On fitting straight lines with the same slope but different intercepts, the six formulations of rubber did not differ in respect of intercept. The slope was estimated as 1.53, implying an exponent n of approximately 2.2.

A plot of ln(HIC) versus ln($A_{max}$) is a good straight line. On fitting straight lines with the same slope but different intercepts, the six formulations of rubber did not differ in respect of intercept. The slope was estimated as 2.40, implying an exponent n of approximately 1.2. See Appendix 5 for a warning about the ln(HIC) vs. ln($A_{max}$) relationship.

## *10.4 Gunatilaka et al. (2004)*

### 10.4.1 Summary of experiments

Gunatilaka et al. (2004) dropped a headform (mass 5.4 kg) on to various playground surfaces. The data to be considered here was read from Figures 4 and 5 of their paper, which refer to tanbark surfaces of various depths. Some of the values of $A_{max}$ and HIC reported by Gunatilaka et al. were high: three of the values of HIC at a tanbark thickness of 4 cm were above 1000, for example.

Values of change of velocity deltaV were not reported. The coefficient of restitution can probably be assumed to be the same (and close to 0) for all surfaces, in which case deltaV is the same for all surfaces, for a given drop height.

### 10.4.2 Results: Change of speed

Consider a particular impact surface, that is, a particular tanbark thickness. For tanbark thicknesses of 4 cm, 8 cm, 15 cm, and 18 cm, there are three or four data points. Results from varying drop height may be summarised as follows.

- For all four tanbark thicknesses, ln($A_{max}$) vs. ln(h) was approximately a straight line (the correlation coefficient was between 0.89 and 0.98), and the regression coefficient was within the predicted range (0 to 1) The estimates ranged from 0.48 to 0.74, and the implied values of n ranged from 0.9 to 2.8.
- For all four tanbark thicknesses, ln(HIC) vs. ln(h) was approximately a straight line (the correlation coefficient was between 0.92 and 0.98), and the regression coefficient was within the predicted range (0.5 to 2) The estimates ranged from 0.97 to 1.58, and the implied values of n ranged from 0.5 to 2.6.
- For all four tanbark thicknesses, ln(HIC) vs. ln($A_{max}$) was close to a straight line (the correlation coefficient exceeded 0.98). The regression coefficient was within the predicted range (greater than 2) for only one of the four tanbark thicknesses. However, in each case the regression coefficient was sufficiently close to 2 that, taking into account its standard error, it was consistent with a value a little greater than 2.

There were only three or four drop heights for each tanbark thickness, and consequently the regression coefficients are imprecisely estimated. For each of the tanbark thicknesses, a value of n can be found such that all three regression coefficients are consistent with it. For example, in the case of tanbark thickness being 8 cm, the hypothesis that n = 6 is not rejected for each of the regressions.

### 10.4.3 Results: Change of surface

Consider a particular drop height. There are data points for between three and seven tanbark thicknesses at each of four drop heights.

- In each case, ln(HIC) vs. ln($A_{max}$) was close to a straight line (the correlation coefficient exceeded 0.99).
- And in each case, the regression coefficient was close to the predicted value of 1.5 (in the range 1.42 to 1.55).

## *10.5 Chang et al. (2004)*

Chang et al. (2004) dropped a headform on to rubber tiles intended for use as playground surface. The data to be considered here was read from Figures 3 and 4 of their paper, which give $A_{max}$ and HIC for five thicknesses of tiles (with between 2 and 9 drop heights each). Chang et al. used a 4.6 kg headform.

Results for a tile thickness of 15 mm may be summarised as follows. When drop height was multiplied by 2 (from 0.3 m to 0.6 m), $A_{max}$ was multiplied by 1.82 and HIC was multiplied by 3.52. Whether n is estimated from ln($A_{max}$) vs. ln(h), from ln(HIC) vs. ln(h), or from ln(HIC) vs. ln($A_{max}$), it is between 5.4 and 7.3.

Results for a tile thickness of 80 mm may be summarised as follows. When drop height was multiplied by 5 (from 0.6 m to 3.0 m), $A_{max}$ was multiplied by 3.00 and HIC was multiplied by 13.25. Whether n is estimated from ln($A_{max}$) vs. ln(h), from ln(HIC) vs. ln(h), or from ln(HIC) vs. ln($A_{max}$), it is between 1.4 and 2.8.

Consider a drop height of 0.6 m. When tile thickness changed from 80 mm to 15 mm, ln($A_{max}$) increased by 1.51, ln(HIC) increased by 2.47, and thus the slope is 1.64, not far from the predicted value of 1.5.

## *10.6 Chang and Huang (2007)*

### 10.6.1 Tests of rubber tiles

A series of tests is sometimes conducted with energy of impact constant. That is, both m and v change, in such a way that ½.m.$v^2$ remains constant. The proportionality relationship in section 4.5 for maximum acceleration may be rearranged as follows:

$$A_{max} \propto (m.v^2/k)^{-1/(n+1)}.v^2$$

Thus for a series of constant-energy tests with the same material, $A_{max}$ will be proportional to $v^2$.

Chang and Huang (2007, Figure 4) gave examples of acceleration pulses obtained in drop tests with rubber composite specimens. Across the range of speeds used by Chang and Huang, $v^2$ changed by a factor of 3.0, and $A_{max}$ changed by a factor of 2.7, not very different.

### 10.6.2 Finite element modelling

Chang and Huang also reported some calculations with a finite element model.

Results for impact with solid rubber tile may be summarised as follows.

- An increase in impact speed from 5.9 to 7.0 m/sec led to $A_{max}$ increasing from 239 to 311 and HIC increasing from 2263 to 3903.
- The result from $A_{max}$ implies that exponent n = 3.4, and that from HIC implies that n = 2.7. These are reasonably consistent.

However, most of the results for impact with rubber tiles described as honeycomb show unusual behaviour of HIC. The strongest example of this occurred with a design referred to as 1:6.

- An increase in impact speed from 5.9 to 7.0 m/sec led to $A_{max}$ increasing from 166 to 204 and HIC increasing from 1247 to 1334.
- The result from $A_{max}$ is not unusual. It implies n = 1.5. The increase of HIC is much less than expected. Even if n were 0, HIC would be expected to increase in proportion to v, that is, by 19 per cent. The small increase of 7 per cent is inconsistent with the theory in section 4.3 of this book.

## *10.7 Ohue and Miyoshi (2014)*

Ohue and Miyoshi (2014) conducted drop tests of a 4.6 kg headform on to four rubber surfaces. They measured both $A_{max}$ and HIC.

Figure 7 of Ohue and Miyoshi shows $A_{max}$ plotted against HIC, for four materials and four drop heights, and a single fitted line implying that $A_{max}$ is proportional to $HIC^{0.44}$.

My opinion (section 6.5 of this book) is that the co-variation of $A_{max}$ and HIC due to drop height should not have been conflated with that due to material.

## *10.8 Eager et al. (2016)*

Eager et al. (2016) dropped a 4.6 kg headform on to specimens of foams and rubbers of thicknesses between 5 cm and 13 cm and having a wide range of impact properties. Ten drops were carried out at each of several drop heights from 0.5 m to 3.25 m. Output variables included $A_{max}$, HIC, length of the time interval over which the integration to calculate HIC was performed, and contact time. Most of the results were presented as plots versus drop height.

Eager et al. report that for one material, when drop height was multiplied by 6, the result was that HIC was multiplied by 26.58, $A_{max}$ was multiplied by 5.67, and contact time was multiplied by 0.67.

- The result for HIC implies an exponent of 1.83 in the proportionality relationship, and thus n = 7.9.

- The result for $A_{max}$ implies an exponent of 0.97 in the proportionality relationship, and thus n = 31.
- Assuming that contact time is proportional to a power function of drop height, the exponent is calculated to be -0.22. From the dependence of T on v in section 4.5.1, this is -(n - 1)/[2.(n + 1)]. Thus n = 2.6.

Thus for the material concerned, the three estimates of n are qualitatively similar, in that they are all well above 1.

For nine materials, Table 2 of Eager et al. reports estimated values of drop height h and some output variables that would correspond to HIC = 1000.

- Section 6.5.5 of this book suggests HIC is proportional to $A_{max}^{1.5}.v$, and so HIC is proportional to $A_{max}^{1.5}.h^{0.5}$.
- Thus if HIC is constant, $A_{max}^{1.5}$ should be proportional to $h^{-0.5}$, and $A_{max}$ should be proportional to $h^{-0.33}$.
- That is closely true of the data in Table 2 of Eager et al.: plotting $\ln(A_{max})$ vs. ln(h), the regression coefficient is estimated to be -0.34.

## *10.9 Kato et al. (2014)*

### 10.9.1 Summary of experiments

The specific orientation of the paper by Kato et al. (2014) was to the simplification of an ASTM procedure for testing playground equipment that involves dropping an instrumented missile on to the ground surface underneath the equipment children play on. Kato et al. compared results of the two procedures, and proposed equations for estimating the result of the conventional procedure from that of the simplified procedure.

The results of Kato et al. (2014) referred to impacts on to bare ground or on to 6 or 10 or 16 cm of sand. Kato et al. used a 4.6 kg missile. I read data from Figures 4 and 5 of their paper, and in a short paper (Hutchinson, 2015a) considered the relationships in the light of the theory in section 4.3 of this book. My findings are summarised below.

### 10.9.2 Results

The following results are for bare ground.

- The dependence of $\ln(A_{max})$ on ln(h) is approximately a straight line, and the slope is estimated to be 0.78, implying that n is about 3.5.
- The dependence of ln(HIC) on ln(h) is approximately a straight line, and the slope is estimated to be 1.63, implying that n is about 3.0.
- The dependence of ln(HIC) on $\ln(A_{max})$ is approximately a straight line, and the slope is estimated to be 2.03, implying that n is very

large. Concerning the relationship between ln(HIC) and ln($A_{max}$), see also Appendix 5.
- A value of n of about 5 or 10 is compatible with all three slopes.

The following results are for thickness of sand = 16 cm.
- The dependence of ln($A_{max}$) on ln(h) is approximately a straight line, and the slope is estimated to be 0.39, implying that n is about 0.6.
- The dependence of ln(HIC) on ln(h) is approximately a straight line, and the slope is estimated to be 1.01, implying that n is about 0.5.
- The dependence of ln(HIC) on ln($A_{max}$) is approximately a straight line, and the slope is estimated to be 2.44, implying that n is about 1.1. Concerning the relationship between ln(HIC) and ln($A_{max}$), see also Appendix 5.
- A value of n of about 0.5 or 1 is compatible with all three slopes.

For the thickness of sand being 6 cm or 10 cm, results are consistent with n being intermediate between the low value found for 16 cm of sand and the high value found for bare ground. However, the standard errors are large.

Consider a particular drop height. There are data points for three or four thicknesses of sand at each of five drop heights. In each case, ln(HIC) vs. ln($A_{max}$) is approximately a straight line. And in each case, the regression coefficient is consistent with the predicted value of 1.5 (in the range 1.46 to 1.95).

## *10.10 Kim et al. (2011)*

### 10.10.1 Introduction

The paper by Kim et al. (2011) is in the Korean language, which I do not understand. I apologise to readers and to Kim et al. if I have misunderstood anything.

The Figures and Tables in the paper are in English. $A_{max}$ and HIC are reported, measured in drop tests of a 3.75 kg head model on to various materials used for playgrounds or building floors. For seven materials, there were three or four drop heights.

### 10.10.2 Results: Variation in drop height

*Dependence of ln($A_{max}$) on ln(h), for each material*. The relationships are approximately straight lines. The relationships in section 4.5.1 imply the slopes should be between 0 and 1. The seven slopes were between 0.53 and 0.99.

*Dependence of ln(HIC) on ln(h), for each material*. The relationships are approximately straight lines. The relationships in section 4.5.1 imply the slopes should be between 0.5 and 2. The seven slopes were between 1.27 and 1.77.

*Dependence of ln(HIC) on ln($A_{max}$), for each material*. The relationships are approximately straight lines. The relationships in section 4.5.1 imply the slopes should be at least 2. Six of the seven slopes were between 2.01 and 2.88, and one was 1.79. Concerning the relationship between ln(HIC) and ln($A_{max}$), see also Appendix 5.

### 10.10.3 Results: Variation in material

*Dependence of ln(HIC) on ln($A_{max}$), drop height of 100 cm*. The scatterplot suggested the result for one material (F-2 in Table 2 of Kim et al.) was an outlier. For the other five materials, the relationship is approximately a straight line. The relationships in section 4.5.1 imply the slope should be 1.5. The slope was estimated to be 1.03.

*Dependence of ln(HIC) on ln($A_{max}$), drop height of 150 cm*. The scatterplot suggested the result for one material (F-2 in Table 2 of Kim et al.) was an outlier. For the other five materials, the relationship is approximately a straight line. The relationships in section 4.5.1 imply the slope should be 1.5. The slope was estimated to be 1.00.

It seems, then, that the several materials cannot be described as varying only in stiffness.

### 10.10.4 Results: Variation in both drop height and material

HIC is predicted to be proportional to $A_{max}^{1.5}.v$ (see sections 6.5.5, 7.8.3, 9.3.3, and 14.3.3). As v is proportional to √h (h = drop height), HIC/√h should be proportional to $A_{max}^{1.5}$. Thus when ln(HIC/√h) is plotted versus ln($A_{max}$), the slope should be 1.5.

When ln(HIC/√h) was plotted versus ln($A_{max}$), with all the surfaces shown, a good straight line was not obtained. That might have been expected from the results in section 10.10.3.

The seven materials fall into groups as follows (see Table 2 of Kim et al.).

- P-1, P-2, P-3. These are playground surfaces, polyolefin foam (30 mm or 50 mm or 70 mm) plus rubber chip (12 mm).
- A-2, A-3. These are building floors, rubber sheet (10 mm or 20 mm) plus PVC sheet (1.8 mm).
- F-1. Building floor, joists span (300 mm) plus PVC sheet (1.8 mm).

- F-2. Building floor, joists span (1000 mm) plus PVC sheet (1.8 mm).

Consider again plotting ln(HIC/√h) versus ln($A_{max}$). A good straight line was found for each of those materials or groups of materials considered separately.

- The slopes were not very different. On fitting lines of a common slope, the common slope was estimated to be 1.39, which is not far from the predicted value of 1.5.
- The failure of the idea of a single straight line of slope approximately 1.5 occurs when putting together data for the four materials or groups of materials. The four intercepts are different, except that the intercept for A-2 and A-3, and that for F-1, are similar.

# 11. Data from impact tests, 5: Adults

There are eleven examples in this chapter. Two are of particular relevance to elderly adults falling, one is about workplace falls, one is about objects falling on to a helmet, two are about protection by aircraft seating, one uses cadaver heads to test padding, and four are about impacts to the foot or leg (though one of those concerns cows, not humans).

As well as those examples, see also section 14.5 for a study relevant to falls of the elderly, in which adult Hybrid III dummies (of two sizes, representing a 5th percentile female and a 50th percentile male) fell from a standing position in five orientations on to a hard surface. Chapters 9 and 10 are on ground impact, in sports and of children.

## *11.1 Fukaya and Uchida (2008)*

### 11.1.1 Summary of experiments

The context of the research reported by Fukaya and Uchida (2008) was injury to elderly people from falling, and the possible usefulness of a wearable airbag. Fukaya and Uchida used a method in which a whole dummy, rather than a headform alone, was dropped in a horizontal posture. The airbag was pre-inflated, as this was not a test of the sensor and inflator. The data to be considered here, obtained from an accelerometer in or on the dummy's head, was read from Figures 8 and 9 of their paper. Drop height is h.

### 11.1.2 Results

Results for falls without a wearable airbag may be summarised as follows. The slope of the dependence of $\ln(A_{max})$ on $\ln(h)$ is 1.06 (slightly outside the predicted range), the slope of the dependence of ln(HIC) on $\ln(h)$ is 1.92 (implying n is about 18), and the slope of the dependence of ln(HIC) on $\ln(A_{max})$ is 1.74 (slightly outside the predicted range). In all these cases, the scatterplots were good straight lines. (See section 10.10.2 for the ranges for the slopes that are possible according to section 4.5.1.)

Results for falls with a wearable airbag may be summarised as follows. The slope of the dependence of $\ln(A_{max})$ on $\ln(h)$ is 1.05 (slightly outside the predicted range), the slope of the dependence of ln(HIC) on $\ln(h)$ is 1.44 (implying n is about 1.7), and the slope of the dependence of ln(HIC) on $\ln(A_{max})$ is 1.29 (outside the predicted range).

Many of the impacts in these experiments were very severe. Without an airbag, $A_{max}$ exceeded 3000 m/sec/sec and HIC exceeded 2000 in the four tests at drop heights of 2 m or greater. Even with an airbag, $A_{max}$ exceeded 3000 m/sec/sec and HIC exceeded 2000 in the two tests at drop heights of 3 m or greater. It may be speculated that the theory of section 4.3 is not applicable to such severe impacts.

## *11.2 Wright and Laing (2012)*

### 11.2.1 Summary of experiments

The context of the research reported by Wright and Laing (2012) was injury to elderly people from falling, and the possible usefulness of novel compliant floors in mitigating impacts.

Wright and Laing reported drop tests of headforms. Their Table 1 gives results for commercial carpet (which is not one of the novel flooring systems). It reports both $A_{max}$ and HIC for three orientations of the headform (front, side, back) at three impact speeds (1.5 m/sec, 2.5 m/sec, 3.5 m/sec).

### 11.2.2 Results

Consider first the effect of changing impact speed. There were only three impact speeds, and the slopes can only be estimated imprecisely.

- For all three orientations of the headform, the slope of the dependence of $\ln(A_{max})$ on $\ln(v)$ was in the predicted range (i.e., between 0 and 2).
- For all three orientations of the headform, the slope of the dependence of ln(HIC) on $\ln(v)$ was in the predicted range (i.e., between 1 and 4).
- For all three orientations of the headform, the slope of the dependence of ln(HIC) on $\ln(A_{max})$ was consistent with being in the predicted range (i.e., greater than 2). It was slightly greater than 2 in one case, and slightly less than 2 in the other two cases. Concerning the relationship between ln(HIC) and $\ln(A_{max})$, see also Appendix 5.

Now consider the dependence of ln(HIC) on $\ln(A_{max})$ as orientation of the headform changes. There were only three orientations, and the slopes can only be estimated imprecisely. If the effect of this were to change stiffness, the slope would be 1.5. For the three impact speeds, the slopes were 0.8, 1.4, and 1.3. All these are compatible with (that is, are not significantly different from) a true slope of 1.5.

## *11.3 Martin et al. (2014)*

The study of Martin et al. (2014) was concerned with firefighter personal escape rope systems. Martin et al. tested three types of rope, a single drop weight (84 kg), and three fall heights (from 20 cm to 42 cm). Maximum loads were reported in their Figure 6. (The maximum for any of the nine combinations of rope and fall height was about 16 kN.)

For the stiffest rope, the increase of fall height by a factor of 2.1 led to an increase of load (measured on the drop weight) by a factor of about 1.37. In terms of the testing described in chapter 2, I presume that the load cell is analogous to the accelerometer inside a headform, and that the rope is analogous to a car's bonnet. In that case, the proportionality relationships of section 4.5 may hold. The exponent n is about 0.7.

For the least stiff rope, the increase of fall height by a factor of 2.1 led to an increase of load by a factor of about 1.23. That corresponds to an exponent n of about 0.4.

## *11.4 Long et al. (2015)*

Long et al. (2015) report results of calculations using a finite element model of an object falling on to a head that is wearing a helmet of the type used in the construction industry. Impact speeds were between 8 m/sec and 18 m/sec, and the object had a mass of either 2 kg (a steel cylinder) or 5 kg (a wood board). Long et al. do not fit any equations to their results.

I have plotted ln(HIC) versus ln(v). For both impacting objects, the plot is a good straight line. The slopes are 3.12 (2 kg steel) and 2.52 (5 kg wood).

It is possible the proportionality relationships of section 4.5 may be relevant in this apparently rather different situation. (A car bonnet that may be represented as a spring is replaced by a hard hat that may be represented as a spring.) If so, the slope of 3.12 would imply than n is about 2.4, and the slope of 2.52 would imply that n is about 1.0.

## *11.5 Saunders et al. (2012)*

### 11.5.1 Introduction

This example might have gone into chapter 7 with other transport examples, or into chapter 12 with military examples.

The context of the paper by Saunders et al. (2012) was the use of visco-elastic polyurethane (VEPU) foams in aircraft seating. They make the point that the great majority of aircraft crashes are considered potentially survivable. For improved survivability, deceleration needs to be spread over a long time and a long distance, and this is achieved by such mechanisms as landing gear stroke, collapse of the sub-floor structure, stroking of the seat structure, and compression of the seat cushion.

According to Saunders et al. (p. 11), "the load limiting function is designed such that it allows the seat and occupant to move (stroke), at loads just under the humanly tolerable limit, over the maximum distance available". The key phrases here are *maximum distance* and *humanly tolerable*.

- A cushion gives only a certain distance in which to change the human's velocity. As described in section 1.7, the term bottoming out is used to refer to the impacted structure (e.g., the foam) deforming so much that it contacts (or almost contacts) some very stiff structure (e.g., a steel block). Acceleration, and hence injury, greatly increases when this occurs. A plot of stress against strain (force against deflection) steepens very much, as in Figure 2 of Saunders et al.
- Unfortunately, however, the phrase *humanly tolerable limit* is vague. Suppose only a certain distance is available, and it is desired to prevent any injury at all from occurring. Minor injury is typically much more common than serious injury, so a relatively soft cushion will be selected that reduces some minor injuries to no injury. There may be little reduction of serious and fatal injuries, because bottoming out will occur. Alternatively, suppose it is desired to prevent fatal injury from occurring. In this case, a relatively stiff cushion will be selected. In low-energy impacts, the stiff cushion will not be as effective as a soft cushion; in effect, it will itself be injurious.
- Thus it seems likely that foams for which stiffness is nonlinear or is velocity-dependent may be advantageous; but appropriate selection of a single intermediate stiffness may also be an improvement on both too soft and too stiff cushions.

Saunders et al. (2012) used a test rig having a mass, with an accelerometer on it, that falls on to the foam. They reported peak (maximum) acceleration and other quantities relevant to the cushioning of impact.

### 11.5.2 Results

Only a subset of results will be discussed. There are some further comments in Hutchinson (2014a). As examples, let us consider maximum accelerations observed with three foams of 3 inches thickness, each at the three speeds of 2.0 m/sec, 2.8 m/sec, and 8.2 m/sec. (Baseline-Yellow is not a VEPU foam, the others are.)

- *Baseline-Yellow*. $A_{max}$ was 20.4 g, 64.9 g, and 898.8 g at the three speeds.
- *Confor Tri*. $A_{max}$ was 11.2 g, 16.7 g, and 202.0 g.
- *Sunmate XX*. $A_{max}$ was 57.3 g, 59.6 g, and 160.1 g.

Of these, Sunmate XX is the least effective cushion at low speed, but is the most effective at the highest speed. I presume this reflects its properties (probably including higher stiffness) preventing or mitigating bottoming out, as described in section 11.5.1.

The theory of section 4.3 suggests summarising the effect of speed on peak acceleration by an exponent. This is 2.n/(n + 1) in terms of the exponent n of section 4.5. Therefore it cannot exceed 2. If it does exceed 2, the conclusion will be that the differential equation of section 4.3 is incorrect unless modified in some way. A possible reason is bottoming out of the foam.

Exponents may be calculated that refer to 2.0 m/sec to 2.8 m/sec and to 2.8 m/sec to 8.2 m/sec. These were as follows.

- *Baseline-Yellow*. 3.44 and 2.45.
- *Confor Tri*. 1.19 and 2.32.
- *Sunmate XX*. 0.12 and 0.92.

Conclusions are therefore as follows.

- *Baseline-Yellow*. Bottoming out may have occurred even at the low speed range of 2.0 m/sec to 2.8 m/sec (and also at the higher speed range 2.8 m/sec to 8.2 m/sec).
- *Confor Tri*. Bottoming out did not occur at the low speed range of 2.0 m/sec to 2.8 m/sec; an exponent of 1.19 implies that n is about 1.5. Bottoming out did occur at the higher speed range.
- *Sunmate XX*. Bottoming out did not occur at either speed range. The two estimates of n are different, suggesting that the theory of section 4.3 is not valid here.

For the Baseline-Yellow foam, the steep increase of $A_{max}$ from 64.9 to 898.8 when speed was increased from 2.8 m/sec to 8.2 m/sec might itself suggest bottoming out. Calculating that the corresponding exponent is 2.45, and knowing that this is incompatible with power-law stiffness, considerably reinforces this interpretation.

### 11.5.3 Discussion of the properties of the foams

In the experimental programme of Saunders et al., several properties of the VEPU foams differed from those of the standard foams. Saunders et al. did not attribute the advantages of the VEPU foams to one difference rather than another. Without further evidence, the advantages of VEPU foams reported by Saunders et al. should not be attributed to the advanced or sophisticated properties (e.g., velocity-dependence) rather than to the simple properties (e.g., stiffness). The Baseline-Yellow foam was crushed by 50 or 60 mm before much deceleration occurred: that seems to be a waste of available distance, and better protection would be given simply by greater stiffness.

## *11.6 DeWeese and Moorcroft (2004)*

DeWeese and Moorcroft (2004) reported work aimed at developing a component test method, as contrasted with a sled test using a complete dummy, for aircraft seating.

Table 1 of DeWeese and Moorcroft gives impact test results at several speeds. HIC is reported, and also a quantity that I will call $A_{av}$. This is the average acceleration during the period of integration for the calculation of HIC. (It seems to be the same as the Skull Fracture Correlate of Vander Vorst et al., 2003, 2004; for this, see section 5.3.) $A_{av}$ will be proportional to $(m/k)^{-1/(n+1)}.v^{2n/(n+1)}$, as $A_{max}$ is.

There were three impacts with thin aluminium sheets at speeds within 5 per cent of 36.7 ft/sec (11.2 m/sec). One impact gave results that were rather different from the other two impacts. (DeWeese and Moorcroft noted that inconsistent performance of the aluminium sheets arose from sensitivity to details of installation.) Comparing the three impacts, a similar message came from both HIC and $A_{av}$.

- HIC ranged from 911 to 2652, and $A_{av}$ ranged from 69 to 121.
- Plotting ln(HIC) versus ln($A_{av}$), the points formed an approximately straight line. (This does not count for much as there were only three points.)
- The slope was 1.93.
- The slope is different from the 1.5 that would be expected if the theory of section 4.3 holds and the variation is due to variation in effective headform mass or in surface stiffness. Possibly the high severity of two of the impacts is relevant to the high slope.

There were eight impacts with polyethylene foam padding (4.5 inches thick, which is 11.4 cm) at speeds from 18.0 to 43.6 ft/sec (5.5 m/sec to 13.3 m/sec).

- Plotting ln(HIC) versus ln(v), the points formed an approximately straight line of slope 2.77. (This figure is different from the 2.90 reported in Figure 6 of DeWeese and Moorcroft. Possibly they used a different method of estimation.) This implies exponent n is 1.4.
- Plotting ln($A_{av}$) versus ln(v), the points formed an approximately straight line of slope 1.17. This implies exponent n is 1.4.
- Plotting ln(HIC) versus ln($A_{av}$), the points formed an approximately straight line of slope 2.34. This implies exponent n is 1.5.

For drop tests of aircraft landing gear, see section 17.10.

## *11.7 Zhang, Yoganandan, and Pintar (2009)*

Zhang, Yoganandan, and Pintar (2009) conducted free-fall experiments with cadaver heads (with scalp, and having the brain replaced with brain simulant). Impacts were parietal-temporal, with one of three padding materials. Accelerometers were attached to the heads. The data reported is averaged over a few heads.

For each head, there were repeated impacts at increasing drop heights until skull fracture occurred. That means that at greater drop heights, there were fewer heads tested. That is, the data refers to a set of heads that changes as impact speed increases.

For the padding material referred to as 40D, the data for the five lowest speeds is the average of the results for the same two heads.

- Plotting ln($A_{max}$) versus ln(v), the points formed an approximately straight line of slope 1.28. This implies exponent n is 1.8.
- Plotting ln(HIC) versus ln(v), the points formed an approximately straight line of slope 2.78. This implies exponent n is 1.5.
- Plotting ln(HIC) versus ln($A_{max}$), the points formed an approximately straight line of slope 2.17. This implies exponent n is 3.0. (See Appendix 5 for a warning about the ln(HIC) vs. ln($A_{max}$) relationship.)

See section 14.1 for drop tests of skulls.

## *11.8 Legs and feet*

### 11.8.1 Edwards et al. (2006)

The concern of Edwards et al. (2006) was with repetitive impacts of the foot and leg, such as occur during running or walking. Edwards et al. instrumented cadaver legs in order to separately measure strain and acceleration of the tibia. The feet were dropped on to a force platform, and so force was measured also. Two drop heights (3 cm and 5 cm) and two masses (9 kg and 11 kg) were used.

I think that strain, acceleration, and force here are analogous to S, $A_{max}$, and m.$A_{max}$ (maximum deformation of the bonnet, maximum acceleration of the headform, and the corresponding force) in the pedestrian headform testing of chapter 2.

Results were as in the paragraphs following.

Bone strain: effect of an increase of drop height by a factor 1.67.
- If exponent n = 1, there would be an increase by a factor 1.29.
- For the lower mass, an increase by a factor 1.49 was observed.
- For the higher mass, an increase by a factor 1.33 was observed.

Bone strain: effect of an increase of mass by a factor 1.22.
- If exponent n = 1, there would be an increase by a factor 1.11.
- For the lower drop height, an increase by a factor 1.40 was observed.
- For the higher drop height, an increase by a factor 1.25 was observed.

Force: effect of an increase of drop height by a factor 1.67.
- If exponent n = 1, there would be an increase by a factor 1.29.
- For the lower mass, an increase by a factor 1.40 was observed.
- For the higher mass, an increase by a factor 1.33 was observed.

Force: effect of an increase of mass by a factor 1.22.
- If exponent n = 1, there would be an increase by a factor 1.11.
- For the lower drop height, an increase by a factor 1.20 was observed.
- For the higher drop height, an increase by a factor 1.14 was observed.

Acceleration: effect of an increase of drop height by a factor 1.67.
- If exponent n = 1, there would be an increase by a factor 1.29.
- For the lower mass, an increase by a factor 1.29 was observed.
- For the higher mass, an increase by a factor 1.25 was observed.

Acceleration: effect of an increase of mass by a factor 1.22.
- If exponent n = 1, there would be a decrease, the factor being 0.90.
- For the lower drop height, there was a decrease, the factor being 0.92.
- For the higher drop height, there was a decrease, the factor being 0.89.

The results are thus mostly compatible with exponent n being approximately 1.

The general tone of the paper by Edwards et al. is that strain is a reasonable reflection of injury, force behaves rather similarly to strain, but acceleration does not, in that the effect of mass is in the opposite direction. A similar point was made in section 5.2.2.

## 11.8.2 Lin et al. (2017)

Lin et al. (2017) created a finite element model of the human heel. They used this to obtain the dependence of both maximum deformation and maximum von Mises stress on the stiffness of the heel pad, the vertical impact speed being 0.7 m/sec. They were aware that for a linear spring, maximum deformation is inversely proportional to the square root of stiffness, and maximum force is proportional to the square root of stiffness.

At pp. 590-591 of their paper, Lin et al. describe how impact force and deformation might both be harmful, by different mechanisms.

- *Impact force*. Lin et al. refer to "cumulative microtrauma", "damage to the musculoskeletal system", and "musculoskeletal pathologies such as degenerative joint diseases, stress fractures of the lower limbs, and low back pain". Lin et al. do not explain what these three types of harm are, or how they are caused by "excessive impact force", or how strong is the evidence that the cause is force (rather than deformation, for example). They do give references, and perhaps such information is given in them.
- *Deformation*. Lin et al. refer to "exaggerated thinning", "increasing of the internal pressure", and "entrapment of the blood vessels and nerves" of the heel pad. Lin et al. do not explain what these three types of harm are, or how they are caused by "excessive deformation", or how strong is the evidence that the cause is deformation (rather than force, for example). They do give a reference, and perhaps such information is given there.

Figure 5(b) of Lin et al. is obtained from their finite element model, and shows that maximum deformation decreases with stiffness, and maximum force increases with stiffness. Lin et al. were not worried by the simplifications in their models, as their purpose was to qualitatively understand the effects of stiffness.

In Figure 5(b) of Lin et al., stiffness k increases by a factor of 8. It can be seen that maximum deformation is multiplied by about 0.75. This implies that exponent n (see section 4.3) is about 6. Also, it can be seen that maximum von Mises stress is multiplied by about 3.3. This implies (if maximum von Mises stress behaves similarly to maximum force) that exponent n is about 0.7.

The estimates of n are both quite different from 1, and thus a linear spring is not a good approximation. And as the two estimates of n are quite different from each other, the more general Hunt and Crossley model (see section 4.3) seems to be not valid.

### 11.8.3 Quenneville and Dunning (2012)

Quenneville and Dunning (2012) were interested in how the responses of dummy legs compare with those of cadaver legs. A particular focus of their work was the MIL-Lx dummy leg, and how suitable it is in testing relevant to anti-vehicle landmine blasts and to high-force road accidents.

Quenneville and Dunning conducted axial impact tests of legs from cadavers, and of two types of impact test dummy. Several impact speeds were used. Force is reported in Figures 2 - 5 of their paper. Examples of what is shown include the following.

- For cadaver legs, in Figure 4 force is approximately proportional to $v^{2.1}$.
- For a Hybrid III leg not wearing a boot, in Figure 5 force is approximately proportional to $v^{2.6}$.
- For a Hybrid III leg wearing a boot, in Figure 5 force is approximately proportional to $v^{1.8}$.

Quenneville and Dunning show the relationships not as power functions, however, but as straight lines or bilinear.

### 11.8.4 Cows

Drop tests were used by Tierney and Thomson (2003) to assess the performance of mattresses used by dairy cows. The concern was with the impact during the lying down movement of a cow.

A mattress of EVA (ethylene vinyl acetate) had similar performance, in respect of maximum acceleration and maximum displacement, whether it was new or three years old.

A rubber-crumb mattress had worse performance when three years old than when new: $A_{max}$ increased and S decreased by about the same factor, as would be the case (see section 4.5.1) if the effect of age was to increase the stiffness parameter k.

There was a distinct contrast between the EVA and rubber-crumb mattresses.

- For the rubber-crumb mattress when new, $A_{max}$ was similar to what it was for the EVA mattress, but S was much greater.
- For the rubber-crumb mattress when old, $A_{max}$ was much greater than for the EVA mattress, but S was about the same.

Thus it appears that the difference between the two materials is not effectively a difference in k. It seems likely that the acceleration pulse was more sharply peaked for the crumb-rubber mattress than for the EVA mattress. If so, the duration of high acceleration was shorter for the crumb-rubber mattress than for the EVA mattress.

# 12. Data from impact tests, 6: Military and security contexts

Concerning blunt injury, many of the concerns of military and police forces are similar to those of employers generally, and the examples in other chapters will be of interest.

The examples in sections 12.1 and 12.2 are similar to some of those in other chapters: they concern padding in helmets. Others are rather different: they include studies relevant to ensuring that less-lethal projectiles (such as "rubber bullets") are unlikely to kill or seriously injure (sections 12.3 - 12.5), and to protecting against blast (section 12.6).

The example in section 11.8.3 might instead have been in this chapter, as a particular focus of the work was the suitability of a particular dummy leg in testing relevant to anti-vehicle landmine blasts. For penetrating injury, see chapters 15 and 16; for penetrating damage, see sections 16.5 and 16.6; for concern with protecting against a range of impact conditions, see section 19.4.5.

## *12.1 Moss et al. (2014), and Hopping et al. (2010)*

Moss et al. (2014) report on drop tests of helmets on to hemispherical anvils, and on corresponding finite element simulations. They compare two helmet pads used in U.S. Army helmets. There were only two impact speeds (3.0 and 4.3 m/sec), and thus it is impossible to say whether there are any power law relationships in the data. Nevertheless, I have calculated exponents n on that basis. In most cases, both $A_{max}$ and HIC were reported. Both test data and simulation data are summarised below.

*Test data, manufacturer = Team Wendy*. From the dependence of $A_{max}$ on v, exponent n is found to be 1.1. From the dependence of HIC on v, exponent n = 0.7. From the dependence of HIC on $A_{max}$, exponent n = 3.7.

*Test data, manufacturer = Oregon Aero*. From the dependence of $A_{max}$ on v, exponent n is found to be 26.

*Simulation, manufacturer = Team Wendy*. From the dependence of $A_{max}$ on v, exponent n = 1.5. From the dependence of HIC on v, exponent n = 2.0. From the dependence of HIC on $A_{max}$, exponent n = 1.0.

*Simulation, manufacturer = Oregon Aero*. From the dependence of $A_{max}$ on v, exponent n = 3.3. From the dependence of HIC on v, exponent n = 6. From the dependence of HIC on $A_{max}$, exponent n = 1.5.

As mentioned earlier, random error can have a big effect on n calculated from the dependence of HIC on $A_{max}$, and this may be the explanation if this estimate of n is not approximately the average of the others: see Appendix 5.

Hopping et al. (2010) report some padding tests and some helmet tests. There were repeated impacts in the padding tests. Average figures for maximum acceleration were presented. Four pad suspension systems were tested at four speeds between 2.1 m/sec and 5.3 m/sec. I have plotted $\ln(A_{max})$ vs. ln(v).

- The relationships for each of the four pad suspension systems were good straight lines.
- The slopes for the four pad suspension systems were about 1.7, 2.1,1.7, and 1.9.
- Estimating a common slope for the four pad sustension systems, the result was 1.9. That would suggest that exponent n is large (10 or 20, perhaps).

## *12.2 Franklyn and Laing (2016)*

Franklyn and Laing (2016) report experiments relevant to a soldier's head striking a vehicle's interior --- specifically, its roof, as might occur if there is a blast underneath the vehicle. Table 2 of Franklyn and Laing gives results for three conditions at impact speeds of both 6 m/sec and 7 m/sec, and for three further conditions at 7 m/sec only. (By condition of impact, I mean whether the headform was wearing a helmet, and whether and with what the surface was padded.) Both $A_{max}$ and a number from which HIC can be calculated are given.

*Results from variation of impact speed*. As there were only two impact speeds, it is impossible to say whether there are any power law relationships in the data. Nevertheless, I have calculated exponents n on that basis. (I assumed the same n, and thus the same slope of ln(HIC) versus $\ln(A_{max})$, for all three conditions of impact.) From the dependence of $A_{max}$ on v, exponent n is found to be 2.1. From the dependence of HIC on v, exponent n = 1.3. The dependence of HIC on $A_{max}$ implies that exponent n is very large: the slope of ln(HIC) versus $\ln(A_{max})$ was estimated to be 2, the lower limit of what is consistent with the proportionality results of section 4.5.1.

*Co-variation of HIC and $A_{max}$, impact speed = 6 m/sec*. The slope of ln(HIC) versus $\ln(A_{max})$ is estimated to be 0.5. Although that appears quite different from the theoretical value of 1.5, there are only three data points; consequently, the estimate is very imprecise and the difference from 1.5 is not statistically significant.

*Co-variation of HIC and $A_{max}$, impact speed = 7 m/sec*. The slope of ln(HIC) versus ln($A_{max}$) is estimated to be 1.4, similar to the theoretical value of 1.5. Although this is interesting, some people may say that it is unlikely that any simple theory of impact will apply to the most severe condition (no helmet on the headform and no padding on the roof), for which $A_{max}$ was over 1100 g and HIC was over 13000.

## *12.3 Oukara et al. (2013)*

Oukara et al. (2013) reported experiments in which four less-lethal projectiles (all with a soft nose and hard plastic body) were shot at a force wall. They were chiefly interested in possible head injuries.

Oukara et al. assumed that maximum force was proportional to a power function of impact speed. For four projectiles, they found exponents between 1.8 and 2.9.

## *12.4 Pavier et al. (2015a)*

A paper of particular interest, because of its concern with what physical quantities are most closely connected with injury, is Pavier et al. (2015a). The context was that of less-lethal projectiles, intended for riot control. Other data about less-lethal projectiles will be considered in section 12.5. For studies conducted in the context of a projectile striking the head of someone playing a sport, see sections 8.5 - 8.7.

Data both from experiments on pigs and from a finite element model were used by Pavier et al. The following discussion is of the results from pigs; for further information on the experiments on pigs, see Prat et al. (2012).

Projectiles were fired at pigs' ribs. Projectiles were of two types, differing in mass (31 g and 62 g), but both being soft-nosed and of the same calibre. There was some variation in impact velocity. Maximum displacement of the rib is the output variable that will be discussed here. There were some direct measures of injury severity: number of ribs broken, maximum number of fractures of a rib, percentage of pulmonary contusion.

The initial idea was that what matters in regard to rib displacement and measures of injury severity is the kinetic energy of the projectile.

- The proportionality relationships found in section 4.5 of this book imply that maximum displacement is a function of kinetic energy.
- Admittedly the situation is different, as in section 4.5 only the surface struck by the headform deforms, whereas in the

experiments being discussed, both the nose of the projectile and the pig's rib deform. Nevertheless, as the projectile nose is completely squashed early in the impact, and maximum rib displacements are quite large (15 to 45 mm), the results of section 4.5 are probably of some relevance.

Contrary to the kinetic energy hypothesis, Pavier et al. found the maximum displacement is more closely related to momentum than to kinetic energy.

The following is a suggestion for retaining the idea that kinetic energy is the important quantity. As in sections 5.4.3 and 6.7.4, suppose a projectile of mass m moving at speed v strikes a stationary mass M (the thoracic wall), and they then move as one body. (That is, there is no rebound.) The size of the projectile, but not its mass, is likely to affect how much mass of the thoracic wall is put into motion.

- Conservation of momentum implies that speed is (m/(m + M)).v.
- Mass m + M moving at speed (m/(m + M)).v now strikes the rib.
- The assumption being made is that deformation is a function of kinetic energy.
- The kinetic energy of mass m + M moving at speed (m/(m + M)).v is proportional to $(m + M).((m/(m + M)).v)^2$.
- That expression is $(m + M)^{-1}.(m.v)^2$. If m is small compared with M, this is approximately $M^{-1}.(m.v)^2$.

In other words, if what really affects rib deformation is kinetic energy, but the projectile needs to put into motion some considerable mass of the thoracic wall, the results may give the appearance that what affects rib deformation is momentum.

Pavier et al. (2015b) reported experiments on porcine cadaver thoracic cages obtained from a meat processing plant. The data in their Figure 17 supported the above conclusion that maximum displacement is more closely related to momentum than to kinetic energy.

Pavier et al. (2015b, Figure 6) reported impact forces on a rigid wall of three types of projectiles, each having a soft foam nose. It appears to me that if the data were to be plotted as ln(maximum force) vs. ln(v), the lines would have slopes of about 2.5. That is too steep to be compatible with the proportionality relationships in section 4.5.

## *12.5 Two examples using the Viscous Criterion and relevant to less-lethal projectiles*

### 12.5.1 The Viscous Criterion

The Viscous Criterion will be discussed in chapter 13, but two examples in which it is used in research on less-lethal projectiles are in this section.

The key quantity in the definition of the Viscous Criterion, $VC_{max}$, is the maximum of the product of deformation and rate of deformation. That is, $VC_{max}$ is proportional to max(x.x'). It is difficult to measure $VC_{max}$ in experiments, but it can readily be calculated in simulations. $VC_{max}$ is used for such situations as a missile (such as a hard ball) striking the thorax of a person. The idea is that local deformation and rate of deformation of the thorax are important, not the acceleration at the centre of gravity.

### 12.5.2 Ballistic Impact Research Laboratory (2011)

A report from the Ballistic Impact Research Laboratory, Wayne State University, in 2011 gave results of tests of ten types of less-lethal kinetic energy munitions. (Two of these were described as flexible batons, and eight as bean bags.)

Each type of projectile was fired 8 or 10 times at a three-rib ballistic impact dummy. Among the quantities measured was the Viscous Criterion, $VC_{max}$. Deformation (displacement) of the rib, needed for calculation of $VC_{max}$, was measured optically.

For most of the types of projectile, the range of impact speeds was quite narrow. For three types, the range was rather wider, the speed of the slowest round being at most 55 per cent of the speed of the fastest.

For those three types, I plotted $\ln(VC_{max})$ versus ln(v). The relationships appeared to be straight lines. The three types had slopes 2.6, 2.5, and 2.4. Using the relationship in section 13.2 of this book, it may be found that a slope of 2.5 corresponds to an exponent n of 0.33.

However, it is questionable whether this interpretation is valid.

- $VC_{max}$ refers to the deformation and rate of deformation of the rib.
- In contrast, the exponent n refers to a single spring that is equivalent to the total system of projectile and rib.
- If the nose of a projectile were very stiff, all the deformation would be of the rib, and n = 0.33 would refer to the rib.

- If both the rib and the nose of a projectile were undamped nonlinear springs with n = 0.33, the system would behave as an undamped nonlinear spring with n = 0.33.

Thus there are some reasons to think that the estimated n is meaningful. But it seems possible that deformation of the rib does not tell us very much at all about deformation of the projectile nose and of the total system. (And if these are not understood, it seems likely that deformation of the rib is not understood, either.)

### 12.5.3 Nsiampa et al. (2012)

Nsiampa et al. (2012) report $VC_{max}$ calculated in numerical simulations (finite element modelling) of a less-lethal projectile striking a person's thorax.

Nsiampa et al. compare two designs of projectile, each having impacts at two speeds that differed by a factor of 2.

- One of the projectiles had mass 140 gm and a rigid nose. The other had mass 41.9 gm and a deformable nose.
- Speeds of impact were such that there were only two impact energies. For the 140 gm projectile, speeds were 20 m/sec and 40 m/sec. For the 41.9 gm projectile, speeds were 37 m/sec and 73 m/sec.

I will discuss normal (90 degree) impacts to the thorax. Nsiampa et al. also considered horizontal impacts, which were not normal to the surface of the thorax.

*Projectile with rigid nose*. When impact speed increased by a factor of 2, $VC_{max}$ increased by a factor of 3.9. If $VC_{max}$ follows the proportionality relationship given in section 13.2, this implies that n = 1.08. This n can be interpreted as referring to the deformation of the thorax, as the projectile itself is described as being rigid.

*Projectile with deformable nose*. When impact speed increased by a factor of 2, $VC_{max}$ increased by a factor of 4.3. If $VC_{max}$ follows the proportionality relationship given in section 13.2, this implies that n = 0.82. As, presumably, both the thorax and the projectile are deforming, it is uncertain whether this n is at all meaningful. As n = 1 represents the linear case, which many people might accept, the fact that n is estimated to be so close to 1 might be interpreted as suggesting that n is 1 for both the thorax and the projectile, and thus for the equivalent single spring.

## 12.5.4 Reasons for one projectile being less injurious than another

This follows on from the discussion in section 12.5.3 of results of Nsiampa et al. (2012). I should straightaway say that concerning injuriousness of the projectile, I am not referring to accuracy, or whether it may ricochet, or the likelihood of skin penetration, or other issues. I am referring only to the following.

- Impact speed.
- Mass.
- Whether the projectile is deformable.

I will assume there are no other features of the projectiles compared by Nsiampa et al. that might make one more injurious than the other, and I will also assume that $VC_{max}$ is a good proxy for injury. As already noted, the data of Nsiampa et al. consisted of output of numerical simulations, not actual experiments. Other examples of attempting to split an effect into two or more components will be in sections 12.6.4 and 16.1.6.

Consider the following two propositions.

(a) That for a given projectile, both mass m and speed v of which can be varied, the kinetic energy of impact ($½.m.v^2$) determines what $VC_{max}$ is.

(b) That $VC_{max}$ is proportional to kinetic energy of impact.

As far as I can see, Nsiampa et al. did not make either of those claims. Nevertheless, they compared the results of impacts of the two projectiles, the impacts having the same (fairly low) energy; and they compared the results of impacts of the two projectiles, the impacts having the same (fairly high) energy. It seems quite likely that readers of Nsiampa et al. may believe (a) or (b), and possible that Nsiampa et al. may themselves have believed (a) or (b).

If the proportionality relationship to be given in section 13.2 below is correct, (a) and (b) are both wrong. The simplest assumption is that n = 1. (In the last two paragraphs of section 12.5.3, n was empirically found to be approximately 1 for the data of Nsiampa et al.) In this case, $VC_{max}$ will be proportional to $m^{0.5}.v^2$.

Exactly what $VC_{max}$ is proportional to is important if we want to know what characteristics of a projectile make it more or less injurious, and how strong their effects are. Both arguments to be given below agree that lower mass and softer nose make a difference. There is a disagreement about relative importance.

Suppose we think that $VC_{max}$ is determined by ½.m.$v^2$. We will reason as follows.

- For equal energies of impact, $VC_{max}$ for the deformable projectile was 62 per cent of that for the rigid projectile (at the lower energy), or was 68 per cent of that for the rigid projectile (at the higher energy). These are quite similar, and average 65 per cent. That is, the effect of the deformable nose is a reduction of about 35 per cent.
- For the higher energy impacts, speed was double that of the lower energy impacts, and thus energy was multiplied by 4. $VC_{max}$ for the rigid projectile was multiplied by 3.9, and $VC_{max}$ for the deformable projectile was multiplied by 4.3. It looks like $VC_{max}$ is not merely determined by kinetic energy of impact, but is proportional to it.
- The deformable projectile has only 30 per cent the mass of the rigid projectile. At a given speed, its kinetic energy is 70 per cent less than that of the rigid projectile. The effect of lower mass will be to reduce $VC_{max}$ by 70 per cent.
- That is, the lower mass had a greater effect than the deformable nose.

I consider those conclusions to be wrong, because the starting point was wrong. We should instead work on the basis that $VC_{max}$ is approximately proportional to $m^{0.5}.v^2$.

- For convenience of calculation in what follows, I will use units of kg (rather than gm) for m and of 100 m/sec (rather than m/sec) for v. Calculate the ratio $VC_{max}/(m^{0.5}.v^2)$. The results are 19.4 (rigid projectile, low energy impact), 6.4 (deformable projectile, low energy impact), 18.9 (rigid projectile, high energy impact), and 7.1 (deformable projectile, high energy impact).
- The average result for the deformable projectile (the average of 6.4 and 7.1) was 35 per cent of the average result for the rigid projectile (the average of 19.4 and 18.9). The effect of the deformable nose is a reduction of about 65 per cent.
- The deformable projectile has only 30 per cent the mass of the rigid projectile. $VC_{max}$ will therefore be multiplied by $0.3^{0.5}$, which is 0.55. Because of lower mass, $VC_{max}$ will be 45 per cent less.
- That is, the deformable nose had a greater effect than the lower mass.

If it is not true that $VC_{max}$ is approximately proportional to $m^{0.5}.v^2$, the conclusion will be wrong. Suppose, for example, that on first contact the projectile puts into motion some mass of the thorax. Projectile mass and speed will be replaced by an effective mass and speed (see section 12.4). It seems likely that the reduction of speed will be greater for the less massive projectile. Consequently the effect of mass itself will have been understated by the reasoning in the previous paragraph, and the effect of deformability overstated.

Direct evidence about separate effects is not available because mass and deformability were linked together in the work of Nsiampa et al.: high mass and rigid together, and low mass and deformable together. Consequently, a chain of reasoning was necessary. I consider the second chain of reasoning to be preferable because of the theoretical support (see section 13.2 below).

It should be admitted that the chain of reasoning originates from very little data. And, of course, it would be much better done by the original authors than by someone like me who relies on the published account. Nevertheless, it may be of some interest as an in-principle demonstration.

For damage to fibre-reinforced composites (rather than injury to people) as a result of being struck by something hard or something soft, see section 17.15.

## *12.6 An example in which there is extra theory: Protection from blast*

### 12.6.1 Summary of experiments

Bass et al. (2005) report on experiments in which a dummy, instrumented with accelerometers and wearing a bomb suit, was exposed to blast. Four different bomb suits (with helmets) were included in the experiments, and two weights of explosive charge.

Both $A_{max}$ and HIC are used as indicators of probable head injury severity. $A_{max}$ and HIC are likely to be correlated, though it was shown in section 6.5 that their relationship depends on what it is that is causing both of them to vary. Some theory to be given in the next section will be applied to data from Bass et al. (2005) and Makris et al. (2000).

### 12.6.2 Theory

The necessary theory is partly that of chapters 4 and 5, and partly that of Bass et al. (2005).

Bass et al. do give some theory for their results, using two explanatory variables, the frontal area of the helmet and the mass of the helmet and head. The theory is as follows. "The acceleration of a head under blast pressure loading is directly related to the frontal projected area of the head or helmet, and acceleration under an applied external force is inversely related to the mass of the head/helmet." (It is evidently assumed that the helmet and head can move relative to the rest of the suit and the wearer's body.) Figure 11 of Bass et al. (2005) shows that HIC for a particular suit is positively related to the ratio area/mass for that suit. Straight line relationships are shown for both charge (explosive) weights.

There are four steps in obtaining consequences of the theory of Bass et al.

- The key components of the theory: force is proportional to area, force is externally determined by the blast pressure, and acceleration = force/mass is true instantaneously.
- The consequence: an increase of area or decrease of mass will increase acceleration at any moment of time by a constant factor, and the acceleration pulse will be linearly stretched in height but unchanged in duration.
- In view of the definition of HIC in terms of acceleration, the natural prediction is that HIC is proportional to $(\text{area/mass})^{2.5}$. Dependence is not linear. Similarly, $A_{max}$ will be proportional to area/mass. ($A_{max}$ was not reported by Bass et al., but is available in other datasets.)
- In addition, if changes in HIC and $A_{max}$ are due to changes in area/mass, HIC will be proportional to $A_{max}^{2.5}$.

Thus if ln(HIC) is plotted versus $\ln(A_{max})$ or versus ln(area/mass), where ln(.) represents the natural logarithm, a straight line of slope 2.5 is expected.

The foregoing proportionality relationships rely upon the acceleration pulse only changing by linearly stretching (or compressing) of height. This may or may not be commonly true for blast.

Bass et al. do not claim that the sentence quoted above is a complete account of blast protection. Observations of the relationships proposed in the earlier paragraphs will have scatter in them, some of this being systematic and due to details of helmet design, and some being random. One of the mechanisms of protection is energy absorption by the helmet (Makris et al., 2000, p. 457), and this is not considered by the theory of Bass et al. The example of Figure 4 of Makris et al. shows that an effective helmet may lengthen the pulse duration (by a factor of about 40 in that example).

The relationship between $A_{max}$ and HIC depends on what it is that is causing both of them to vary (see also section 6.5). Some causes, such as change in the area-to-mass ratio, may lead to an approximately constant pulse shape with linear stretching (or compression) of acceleration only, and thus an exponent of 2.5. Others, such as energy absorption, may not.

### 12.6.3 Results

In view of the above, three questions will be examined. Firstly, when the area-to-mass ratio is known, is HIC proportional to $(\text{area/mass})^{2.5}$

(across helmets, for a given charge of explosive)? Secondly, when the area-to-mass ratio is not available, is HIC proportional to $A_{max}^{2.5}$ for (a) a given helmet condition and different weights of explosive and (b) a given weight of explosive and different helmets?

*Dependence of HIC on area/mass.* Taking logarithms of HIC and area/mass, the data points from Bass et al. (Figures 6 and 11) form approximately straight lines for both the heavier and the lighter charge of explosive. The regression estimates of the slope are 2.6 for the heavier charge, and 3.3 for the lighter charge, approximately equal to the theoretical value of 2.5. (There are only four data points, and the slopes are estimated only imprecisely: the standard errors are 0.8 and 1.6.)

*Co-variation of $A_{max}$ and HIC because of explosive charge.* For two helmet conditions, Table 2 of Makris et al. gives data on how $A_{max}$ and HIC change when weight of explosive changes. In the case of there being no helmet, ln(HIC) versus ln($A_{max}$) is close to a straight line, and the slope is 2.49 (standard error = 0.17). For a PASGT helmet without visor, ln(HIC) versus ln($A_{max}$) is close to a straight line, and the slope is 2.65 (standard error = 0.27). (The PASGT helmet without visor was not effective in reducing acceleration.) Thus the slope is consistent with a value of 2.5 in both cases. However, only three charges of explosive were used, so this evidence is quite limited.

*Co-variation of $A_{max}$ and HIC because of helmet design.* Table 1 of Makris et al. gives data on how $A_{max}$ and HIC change when design of helmet changes (and explosive charge is constant). In this case, ln(HIC) versus ln($A_{max}$) is close to a straight line, and the slope is 1.58 (standard error = 0.11). Also, Figures 2(b) and 4 of Makris et al. show that a helmet that reduces $A_{max}$ sometimes greatly lengthens the time that the acceleration pulse lasts. HIC takes into account the time that acceleration lasts, but $A_{max}$ does not. This is reflected in the reduction of HIC being less steep than $A_{max}^{2.5}$.

### 12.6.4 Discussion

*An application of theory: Components of effectiveness.* For a given blast, HIC is an overall measure of how effective a suit (or its helmet) is (and so is $A_{max}$). Observed effectiveness may be split into the area-to-mass ratio, other aspects of helmet design, and random error. If the argument here, based on Bass et al. (2005), is accepted, the ratio HIC/(area/mass)$^{2.5}$ should be used to extract the area/mass effect from HIC. (Also, the ratio $A_{max}$/(area/mass) could be used similarly.)

- For the higher charge of explosive in Figure 6 of Bass et al., the observed HIC places the four suits in the order (from most to least effective) D, A, B, C.
- The ratio area/mass places the suits in the order A, D, B, C.

- The ratio of HIC/(area/mass)$^{2.5}$ places the suits in the order D, C, A, B.

Of course, these orderings (a) are subject to random error, and (b) reflect HIC only rather than any other aspect of injury. (Other examples of attempting to split an effect into two or more components are in sections 12.5.4 and 16.1.6.)

*A comment on lengthening of the acceleration pulse.* In the comparison of different helmets by Makris et al., HIC was not proportional to $A_{max}^{2.5}$. It may be that the ratio area/mass did not vary so much from one suit to another in the study of Makris et al. as it did in that of Bass et al. Instead, variation between helmets in respect of energy absorption may have been responsible for the lengthening of the acceleration pulse and the co-variation of $A_{max}$ and HIC. (The two cases of an exponent close to 2.5 when charge of explosive varied referred to no helmet and to the PASGT helmet without visor.) From a baseline of 0 per cent reduction (no helmet), the results in Makris et al. extended to 92 per cent reduction in peak acceleration $A_{max}$ and 98 per cent in HIC. This is a very wide range, and thus the empirical result (an exponent of about 1.58) has some importance, even without theoretical backing.

*Maximum acceleration and HIC.* In the context of blast injury, Dionne et al. (2010) present a graph (their Figure 10) showing dependence of HIC on $A_{max}$. (They refer to acceleration, but presumably they mean maximum acceleration.) Dionne et al. make clear the curve they present is empirical, but they do not make clear whether there is also theoretical basis for it. They do not give a formula, but the curve is very close to HIC $\propto A_{max}^{2.5}$. They do not state what variable is presumed to be causing the co-variation of $A_{max}$ and HIC.

Data description and analysis often benefit from being guided by theory. For experiments on blast and on blunt impact, some theory is now available. Theory may suggest relationships of dependent variables (e.g., HIC and $A_{max}$) to quantitative independent variables (e.g., the ratio area/mass) and also inter-relationships of dependent variables. Specifically, the relationship between $A_{max}$ and HIC will depend on what it is that is causing both of them to vary.

# 13. Data from impact tests, 7: The Viscous Criterion

## *13.1 Introduction*

Two examples of the use of the Viscous Criterion were given in section 12.5, the context being less-lethal projectiles, as used by security forces in riot control.

The key quantity in the definition of the Viscous Criterion, $VC_{max}$, is the maximum of the product of deformation and rate of deformation. That is, $VC_{max}$ is proportional to max(x.x'). There is also division by the initial thickness of what is being compressed (e.g., the human thorax). See Lau and Viano (1986). (It appears that these authors used Viscous Response for what has become known as the Viscous Criterion.) It is difficult to measure $VC_{max}$ in experiments, but it can readily be calculated in simulations. Some properties are given by Wang (1989).

Newman (1987) considered that $VC_{max}$ has great intuitive appeal. "It is consistent with the simple notion that if you compress the chest enough, ribs will break. And if you do it fast enough, the intrathoracic organs don't have time to get out of the way, and they too can be injured."

$VC_{max}$ is used for such situations as a missile (such as a hard ball) striking the thorax of a person, with there being deformation of the thorax and possibly consequent injury. The context envisaged in chapter 4 was that of a rigid body (representing a pedestrian's head) striking a yielding surface (such as the bonnet of a car). These contexts differ, but there are similarities: in both, the striking missile is of low mass and is regarded as rigid, the struck object is of high mass and deforms, movement can be modelled with a differential equation in deformation x, and the relevant output (reflecting injury) is determined by the acceleration pulse.

In the pedestrian case, the potential for injury is measured by the acceleration of the rigid headform. In the missile impact case, injury is thought to arise because of the fast deformation of the human. Among the implications is that this leads to higher m implying higher $VC_{max}$ and injury in the missile impact case. This might be considered a warning against applying the proportionality relationships in section 4.5 without thought. (The mass m refers to the low-mass object being accelerated, which is the human's head for pedestrian impact but is the ball for missile impact. This leads to higher m implying lower acceleration in the pedestrian impact case.)

## *13.2 Theory based on a differential equation*

In chapter 4 and here, it is envisaged that immediately after the moment of first contact, the surface of the thorax moves at speed v. Thus, in effect, there is an assumption that any mass put in motion is negligible compared with the mass of the missile. The change from being stationary is assumed to be instantaneous, and whatever force, acceleration, and injury occurs at the moment of first contact is regarded as outside the scope of the theory. The key point is that deformation x is measured from what it is at the moment when x' = v (that is, x(0) = 0 when x'(0) = v). A possible generalisation would be to replace m by m + M, and v by (m/(m + M)).v, where M is an unknown mass needing to be fitted to data; for this, see sections 5.4.3, 6.7.4, and 12.4.

$VC_{max}$ was not one of the output variables considered in section 4.5. However, similar reasoning applies.

- An important property of the differential equation in section 4.3 is that the shape of the acceleration pulse x'' remains constant, except for linear stretching (vertically and/or horizontally), if m or v change. Thus the shapes of x and x' remain constant, except for linear stretching.
- Consider some particular acceleration pulse y''(t) and the linearly stretched pulse A.y''(t/T). These are respectively the second differentials of deformations y(t) and $A.T^2.y(t/T)$. The rates of change of deformation are y'(t) and A.T.y'(t/T).
- The product of deformation and rate of deformation is y(t).y'(t) in the first case, and $A^2.T^3.y(t/T).y'(t/T)$ in the second case. Its maximum in the first case is therefore multiplied by $A^2.T^3$ in the second case.
- In terms of m and v, the stretching factors A and T are $A = m^{-1/(n+1)}.v^{2n/(n+1)}$ and $T = m^{1/(n+1)}.v^{-(n-1)/(n+1)}$. This may be seen in equation (5) of Hutchinson (2013) and in section 4.4 of this book. The general approach in obtaining A and T in terms of m and v was to suppose that y(t) is a solution of the differential equation, and then show that a linear stretching is also a solution. That is, suppose that y satisfies the three requirements $y'' + y^n.[1 + b.y'] = 0$, y'(0) = 1, and y(0) = 0; then $A.T^2.y(t/T)$ satisfies the differential equation in section 4.3, y'(0) = v, and y(0) = 0.
- Thus the effect on max(y.y') is to multiply it by $A^2.T^3 = m^{1/(n+1)}.v^{(n+3)/(n+1)}$. It is concluded that $VC_{max}$ is proportional to $m^{1/(n+1)}.v^{(n+3)/(n+1)}$. Another way of expressing this is that $VC_{max}$ is proportional to $E^{1/(n+1)}.v$, where E is the kinetic energy of the impacting body.
- If, for example, n is 1, $VC_{max}$ is proportional to $m^{0.5}.v^2$.

As in section 4.5, m should be interpreted as m/k.

It is quite likely that both the human and the inanimate object that strikes, or is struck by, the human will deform. If both deform, deformation and rate of deformation of the human are the quantities of interest. Furthermore, a distinction needs to be made between the stiffnesses of the human and of the object. The next paragraph will sketch how the term in stiffness is modified if both the human and the inanimate striking object deform.

Some consideration was given in section 5.3 to the possibility that the human deforms, and that it may be more appropriate to describe this (rather than acceleration, or force) as the reason for injury. In section 5.3, deformation of the human ($x_{2max}$) was $(k/k_2)^{1/n}$ multiplied by total deformation (S). In the context of $VC_{max}$, the multiplying factor will need to be applied to deformation and to rate of deformation. That is, the product $x_2.x_2'$ will be $(k/k_2)^{2/n}$ multiplied by the maximum of x.x'. As this is proportional to $k^{-1/(n+1)}$ (and terms in m and v), the maximum $x_2.x_2'$ will be proportional to $k^{[-1/(n+1)]+(2/n)}$ (and terms in $k_2$, m, and v). Thus $VC_{max}$ is proportional to $k^{(n+2)/[n.(n+1)]}$.

$VC_{max}$ has usually been calculated using a springs-and-dampers mathematical model, or a finite element model. It is not to be expected that the differential equation will exactly represent the behaviour of these, and there are even greater levels of difficulty in respect to physical models, animals, and cadavers. Nevertheless, it is worth making comparisons of the predictions with data.

## *13.3 Initial contact*

Section 5.4 raised the possibility that some special phenomenon occurs at first contact: perhaps low stiffness (and hence low force and low acceleration), or extra energy absorption, or putting a mass into motion. Such phenomena would distort $VC_{max}$, as well as the output variables considered in section 4.5.

Section 12.4 has some discussion of putting a mass into motion in the context of rib injury. $VC_{max}$ was not used there, but it could have been.

Using a springs-and-dampers model, Lau and Viano (1986, Figures 13 and 14) show that the rise in speed after first contact is not instantaneous but takes several milliseconds, and the maximum speed is less than v.

## *13.4 Lau and Viano (1986)*

Lau and Viano (1986) set up a thorax model consisting of springs and dampers, and simulated impacts having two combinations of mass and speed, energy being the same (that is, mass of the impactor was inversely proportional to square of speed). Two aspects of the results are of particular interest.

If the relationship given in section 13.2 holds, then when impact energy is constant $VC_{max}$ will be proportional to v, whatever n is. Lau and Viano found that when v changed from 7 m/sec to 15 m/sec, $VC_{max}$ changed from 0.8 m/sec to 1.7 m/sec. However, the range over which $VC_{max}$ is proportional to v is quite narrow. Indeed, Figure 15 of Lau and Viano shows $VC_{max}$ increasing at first with v, and then decreasing.

## *13.5 Wang (1989)*

Wang (1989) used a similar method. There were three simulated impacts of the same energy. Two predictions may be made.

Wang's equation (46) gives a proportionality relationship for what is termed the reduced form of the Energy Storing Rate Criterion. This is very similar to $VC_{max}$. The result given in section 13.2 implies that the ratio of the $VC_{max}$ to v will be constant. Calculations of the ratio from Wang's Figure 7 show this to be so (0.108, 0.112, and 0.113 in the three impacts).

Wang's equation (45) gives a proportionality relationship for what is termed the reduced form of the Stored Energy Criterion. This depends only on maximum deformation. The proportionality relationship given in section 4.5 shows that maximum deformation is proportional to a power function of impact energy. As impact energy is constant in Wang's three impacts, the Stored Energy Criterion should be the same in all three impacts. Wang's Table 1 shows this to be so (relative values of 1.00, 1.05, and 0.96).

## *13.6 Lau et al. (1987)*

### 13.6.1 Summary of experiments

Lau et al. (1987) conducted experiments in which the abdomens of swine struck the lower rim of either a stiff or a softer steering wheel. (See also Horsch et al., 1985.) Both $VC_{max}$ and maximum compression of the abdomen were reported. There was only one speed of impact, and one mass of swine.

### 13.6.2 Theory

The stiffnesses of the systems of abdomen plus a steering wheel were not stated. Hence a prediction cannot be made on that basis. But as two possible proxies for injury were given ($VC_{max}$ and maximum compression), their co-variation can be considered.

Human deformation $x_{2max}$ was found in section 5.2.2 to be proportional to $k^{1/[n.(n+1)]}$, where k is the equivalent stiffness of the system consisting of the (possibly deformable) inanimate object that strikes, or is struck by, a (possibly deformable) human.

Similarly, $VC_{max}$ was found in section 13.2 to be proportional to $k^{(n+2)/[n.(n+1)]}$.

Thus if k changes, and $k_2$, m, and v do not change, $x_{2max}$ and $VC_{max}$ will be expected to co-vary. $VC_{max}$ will be proportional to $x_{2max}^{n+2}$.

### 13.6.3 Results

The results considered here are the respective averages for the stiff and the softer steering wheel.

The ratio of compression $x_{2max}$ for the two steering wheels was 1.24 (stiffer divided by softer).

The ratio of $VC_{max}$ for the two steering wheels was 2.09 (stiffer divided by softer).

Consequently, n is estimated to be about 1.5.

## *13.7 Impacts of robots with humans*

### 13.7.1 Park et al. (2015)

Park et al. (2015) modelled impacts of robots with humans. They used a springs-and-dampers model to simulate values of several proxies for injury at several speeds. Proportionality between force and displacement is evident in their equation (7). It might be that the complexities of the collision model have relatively minor effects. If so, it will be expected that the proportionality relationships given earlier will hold with n = 1. That is, $VC_{max}$ will be proportional to $v^2$.

Table X of Park et al. shows that when v changes from 1.0 to 2.0, $VC_{max}$ changes from 0.03 to 0.13, approximately what is expected. Results for other proxies for injury --- maximum force and maximum deformation in the case of the chest, and maximum force and HIC in the case of the head --- are also compatible with the relevant proportionality relationship with n being 1.

### 13.7.2 Gao and Irwin (2015)

Discussion of Gao and Irwin (2015) does not properly belong in this chapter, as they did not calculate $VC_{max}$. However, they might have done, as the subject of their interest was robot impact with a human chest, and I will now comment on their results. Gao and Irwin report experiments in which three impactors struck the chest of a Hybrid III dummy (and they make some comments relevant to its biofidelity). Instead of $VC_{max}$, they report maximum chest deformation (S).

Consider a plot of S vs. v. This is likely to be judged by eye to be a set of three linear proportionality relationships. That is, each is a straight line, and in each case S is 0 when v is 0. Such relationships correspond to slopes of 1 when plotting ln(S) vs. ln(v).

Now consider the dependence of ln(S) on ln(v).

- For each of the three impactors, there is a good straight line, and the lines are parallel.
- One of the impactors was of different diameter (d) to the others. Regressing ln(S) on ln(v), ln(m), and ln(d), the first two coefficients were estimated to be 1.27 and 0.72, and the coefficient of ln(d) was not statistically significant.
- According to the relationship in section 4.5.1, $S \propto m^{1/(n+1)}.v^{2/(n+1)}$. If this holds, the coefficient of ln(v) will be twice that of ln(m). Comparing 1.27 with 0.72, we see that is approximately the case; and n is estimated to be about 0.51.
- As a plot of S vs. v shows approximate proportionality, taking logarithms, doing a regression, and finding a slope of 1.27 rather than exactly 1 may not seem very important. But 1.27 corresponds to n being about 0.6, which is rather different from the simplest model in which n is 1. And knowing that the effects of v and m are approximately consistent with each other and with n being about 0.5 is a further reason for thinking we are missing something if we omit to plot ln(S) vs. ln(v).

### 13.7.3 Fujikawa et al. (2017)

Fujikawa et al. (2017) also did not calculate $VC_{max}$. Fujikawa et al. considered that data on minor injury is important in the context of robots working with humans, and they studied the bruising of pigs. They give data for vertical impacts on the chests of three pigs, and for horizontal impacts on the chests of three more pigs. Velocity of impact was varied, and impactor mass also in the case of four pigs. Maximum contact pressure is reported; this is proportional to maximum force, which was measured by a transducer.

I have regressed ln(maximum pressure) on ln(v) and ln(m), with a different intercept for each pig (but the same slopes). The results imply that $F_{max}$ is proportional to $m^{0.4}.v^{0.9}$. This is very close to what would be expected if exponent n were 1.0 (which would be $m^{0.5}.v$).

That may seem a simple and straightforward result, but it is only fair to add that the amount of variation in the data is such that the 95 per cent confidence intervals are wide: from 0.1 to 0.8 for the exponent of m, and from 0.6 to 1.2 for the exponent of v.

### 13.7.4 Cross-reference

Another paper relevant to robot impacts with humans will be discussed in section 14.6.

## *13.8 A different proxy for injury: Thota et al. (2015)*

What is considered here is the ratio of the square of maximum deformation to the pulse duration.

Let S be maximum deformation, $T_2$ be time to maximum deformation, and consider the ratio $S^2/T_2$. (For a linear spring, $VC_{max}$ and $S^2/T_2$ are proportional, but this may not extend to other conditions.) Hutchinson (2013) did not obtain the dependence of $S^2/T_2$ on v, but did find that S is proportional to $m^{1/(c+1)}.v^{2/(c+1)}$; and $T_2$, like pulse duration, is proportional to $m^{1/(n+1)}.v^{-(n-1)/(n+1)}$. It thus follows that $S^2/T_2$ is proportional to $m^{1/(n+1)}.v^{(n+3)/(n+1)}$. This is the same as found in section 13.2 for $VC_{max}$.

Thota et al. (2015) refer to the Viscous Criterion as a factor multiplied by $S^2/T_2$. As far as I know, this is not at the present time a common usage of the term: in principle, it differs from the definition in terms of the maximum of the product of deformation and rate of deformation. It is likely that in many sets of experiments $VC_{max}$ and $S^2/T_2$ will be proportional: as shown here, change of v will change both by the same factor, change of m will change both by the same factor, and change of k will change both by the same factor. (It might be that change of the impacting bodies will be adequately represented by a change of k, with b and n being unchanged.) The case against a simplified concept is that even if in many circumstances there is a good correlation between it and a more difficult concept, there may be occasions when it is not valid.

Thota et al. (2015) considered the deformation of the thorax when subject to blunt impact (e.g., impact of a baseball or a cricket ball). The method used was finite element modelling. Figure 11 of Thota et al. shows the dependence on impact speed of a quantity proportional to $S^2/T_2$. Thota et al. did indeed regard the empirical relationship as a power function, as suggested on theoretical grounds above. They found that the exponent was 2.55. The present interpretation of these findings is that the differential equation approximately holds for the model employed, with $n = 0.29$.

# 14. Data from impact tests, 8: Rigid surfaces

There are eleven examples in this chapter.

## *14.1 Masuzawa et al. (1971)*

Masazawa et al. (1971, Table 2-1) reported drop tests of seven dry human skulls from various heights. There were approximately four frontal impact tests per skull at increasing drop heights, the final one usually having fracture. The data include the drop height, the maximum acceleration (measured with an accelerometer), maximum force (measured with a load cell), and whether the skull fractured or not. One skull has to be excluded as it had only one impact without fracture; of the others, two struck a wooden board, two struck a scalp simulator, and two struck a scalp simulator while wearing a helmet.

For most skulls, maximum acceleration and maximum force are approximately proportional.

I have plotted ln($A_{max}$) and ln(maximum force) versus ln(drop height). The slope is steeper than 0.5 (that is, it is inconsistent with a linear spring), but it is not steeper than 1 (that is, it is consistent with a higher value of the spring exponent). There can be reasonable confidence in that statement as referring to an average slope for the seven skulls. For individual skulls, there are few observations and consequently a lot of uncertainty.

See section 11.7 for drop tests of cadaver heads.

## *14.2 Mertz (1985), and Prange et al. (2004)*

Mertz (1985) reported $A_{max}$ for impacts of five headforms with a rigid surface, at various speeds. (Mertz was chiefly concerned with comparison of the headform results with cadaver results.) There were 32 impacts. Mertz showed $A_{max}$ plotted versus $v^2$; curves were drawn for which $A_{max}$ = 0 at $v^2$ = 0 and that were concave downwards. That suggests $A_{max}$ may be approximately a power function of v, with the exponent being less than 2.

For each headform, I find there is an approximately linear relationship between ln($A_{max}$) and ln(v). The slopes are between 1.1 and 1.7. These values respectively correspond to the exponent n being about 1.2 and 6.

Prange et al. (2004) reported drop tests on to a flat anvil of infant cadaver heads and of a crash test dummy representing an infant (the

CRABI dummy). There was a force sensor, and acceleration was calculated from force.

There were three cadaver heads, each receiving impacts at five locations from two drop heights.

- Plotting ln($A_{max}$) vs. ln(h), the slope was found to be 0.66, which implies exponent n is about 2.0.
- Plotting ln(HIC) vs. ln(h), the slope was found to be 1.49, which implies exponent n is about 1.9.

Those slopes are, in effect, averages over the 15 combinations of head and impact location. The two estimates of n are very similar.

Results for the CRABI head were as follows.

- Plotting ln($A_{max}$) vs. ln(h), the slope was found to be 0.90, which implies exponent n is about 9.
- Plotting ln(HIC) vs. ln(h), the slope was found to be 1.72, which implies exponent n is about 4.

Those slopes are, in effect, averages over the five impact locations. The two estimates of n are qualitatively similar, in that they are substantially greater than 1.

## *14.3 Cripton et al. (2014)*

### 14.3.1 Summary of experiments

Cripton et al. (2014) dropped a Hybrid III headform, either wearing or not wearing a bicycle helmet, on to a flat steel anvil. The helmet had a shell and a liner of expanded polystyrene. Drop heights were between 0.5 m and 3 m. HIC and $A_{max}$ were measured.

Two features of the data should be noted.

- For the headform without helmet, HIC and $A_{max}$ were very high: at the drop heights used, HIC was between 1000 and 17000, and $A_{max}$ was between 200 g and 1100 g.
- Cripton et al. give HIC and $A_{max}$ for various drop heights, and impact speed is not given. Impact speed will be approximately proportional to the square root of drop height. It is not clear from the text (Cripton et al., p. 3) how much error there will be in this.

Presumably the headform, not the steel anvil, deformed. For the acceleration to reflect the acceleration of a human head, the deformation characteristics of the headform would need to be similar to those of a human head. Cripton et al. say that the Hybrid III headform has been validated for bare head impacts.

Mertz (1985) compared five headforms with cadaver results, and said (in the Abstract) that the "Hybrid III response was the most representative". However, in the experiments of Cripton et al., the greatest $A_{max}$ was about 1000 g at about 8 m/sec, whereas in those of Mertz, the greatest $A_{max}$ was about 250 g at about 3 m/sec, so this evidence of validity or biofidelity is rather limited.

### 14.3.2 Results

After taking logarithms of drop height, HIC, and $A_{max}$, one would expect the scatterplots to be straight lines.

- For the helmeted condition, the visual appearance was indeed of straight lines. The n's implied by the ln(HIC) and ln($A_{max}$) scatterplots were 1.0 and 1.3. These are similar, in that they are both approximately 1. However, their standard errors indicate that the difference between them is statistically significant.
- For the helmeted condition, the visual appearance of the scatterplot of ln(HIC) versus ln($A_{max}$) was of a straight line. Exponent n was about 2.8.
- For the no helmet condition, the visual appearance was indeed of straight lines. The n's implied by the ln(HIC) and ln($A_{max}$) scatterplots were similar, in that both were appreciably greater than 1 (2.7 and 3.6).
- For the no helmet condition, the visual appearance of the scatterplot of ln(HIC) versus ln($A_{max}$) was of a straight line. Exponent n was about 14.

### 14.3.3 Interpretation

I think the helmeted and no helmet conditions should be considered separately.

For the no helmet condition, some of the impacts were so severe (that is, HIC and $A_{max}$ were so high) that I doubt if anyone knows the exact implications.

As to the helmeted condition, this is a much more straightforward dataset. However, I am unsure what message to read in it. I will give three.

*One interpretation*. The exponent n seems to be about 1. This is indicated by the effects of drop height on both HIC and $A_{max}$. The larger n implied by the co-variation of HIC and $A_{max}$ is a minor puzzle, but is probably a chance result. Appendix 5 gives reason why the estimate of n may be a poor one.

*A second interpretation*. When there is a discrepancy, a possible explanation is that some phenomenon occurs very early in the impact that necessitates use of the concept of an effective impact speed. In addition, for this dataset, the authors do not seem to be fully confident of the proposition that impact speed is proportional to the square root of drop height. In view of these considerations, the most credible value of n is that from the co-variation of HIC and $A_{max}$, which was 2.8.

*A third interpretation*. If pulse shape remains constant, except for linear stretching or compression vertically and horizontally, HIC is proportional to $A_{max}^{1.5}$.deltaV, where deltaV is the change of velocity. (If the helmeted headform bounces, deltaV will exceed the impact velocity.) Consequently, deltaV is proportional to the ratio HIC/$A_{max}^{1.5}$. For this relationship, see also sections 6.5.5, 7.8.3, 9.3.3, and 10.10.4.

- When the ratio HIC/$A_{max}^{1.5}$ is plotted (vertically) versus the square root of drop height (horizontally), the relationship is seen to be approximately a straight line, with positive intercept on the vertical axis. (I mean, HIC/$A_{max}^{1.5}$ is positive at zero drop height.)
- Thus (HIC/$A_{max}^{1.5}$)/v decreases with v.
- As (HIC/$A_{max}^{1.5}$)/v is proportional to 1+R, where R is the coefficient of restitution, a negative relationship between coefficient of restitution and impact speed would explain the pattern of results.

## *14.4 Some further papers relevant to bicycle helmets*

### 14.4.1 Stalnaker et al. (1992)

Stalnaker et al. (1992) reported drop tests of a Hybrid II headform, either wearing or not wearing a bicycle helmet, on to a flat steel plate.

Results are given for five commercially available and one prototype helmet, and for the headform without a helmet.

Having just discussed the results of Cripton et al. (2014) in section 14.3, I will summarise only the results for a headform without a helmet, in frontal impacts.

- Plotting the logarithms of $A_{max}$ and HIC versus the logarithm of drop height, the relationships were good straight lines.
- For $A_{max}$, the slope was 0.68, which corresponds to n = 2.2.
- For HIC the slope was 1.48, which corresponds to n = 1.9.
- The visual appearance of ln(HIC) versus ln($A_{max}$) was of a straight line. The slope was 2.16, which corresponds to n = 3.2.

### 14.4.2 Warren et al. (2017)

Warren et al. (2017) consider that wearing a bicycle helmet when up a ladder might be an easy protective measure at home. They conducted drop tests of headforms with and without helmets on to a rigid surface. Drop heights of up to 2.5 m were used with a helmet, and up to 0.6 m without a helmet (to avoid damage to the accelerometer). HIC was used as the proxy for injury.

Wearing a helmet makes a tremendous difference to HIC. Table 1 of Warren et al. shows that at a given drop height, HIC is approximately multiplied by 10 when a helmet is not worn. For a given HIC, the drop height without a helmet is approximately one fifth that with a helmet.

Drops were performed with the headform in one of three orientations.

- With a helmet, occipital impacts had lower HIC than frontal or parieto-temporal impacts.
- Without a helmet, there was no consistent difference.

I have plotted ln(HIC) versus ln(drop height).

- With a helmet, the relationship for each orientation appeared to be an approximate straight line. The slopes were very similar, between 1.34 and 1.41. That corresponds to n being between 1.3 and 1.5.
- Without a helmet, the relationship for each orientation appeared to be an approximate straight line. The slopes were between 1.51 and 1.95, corresponding to n being between 2 and 30.

### 14.4.3 Oikawa et al. (2017)

Oikawa et al. (2017) report results from finite element (FE) simulations of a head, either wearing or not wearing a bicycle helmet, falling on to a rigid surface. Several output variables, that may readily be calculated by the FE method, but not measured in an experiment, were considered.

A bicycle helmet succeeded in substantially reducing all the proxies for injury.

The effect of speed on peak skull strain for two angles of impact may be summarised as follows.

- Cases A-(1) and A-(2), no helmet. When vertical velocity changed by a factor of 1.5, strain changed by a factor of 1.3. If a power function relationship is assumed, the exponent is 0.59.
- Cases B-(2) and B-(3), no helmet. When vertical velocity changed by a factor of 1.6, strain changed by a factor of 1.3. If a power function relationship is assumed, the exponent is 0.55.

If it is thought that a power function dependence on v is similar to a power function dependence of S on v (see section 5.2.2), those exponents are interpreted as 2/(n + 1). That is, n is 2.4 and 2.7 in the two cases.

### 14.4.4 Kurt et al. (2017)

Kurt et al. (2017) conducted drop tests of bicycle helmets, both conventional (expanded polystyrene, EPS) and novel (airbag). The airbag helmet was pre-inflated to one of three pressures.

An airbag helmet offers approximately 12 cm of deceleration distance. It is not surprising that $A_{max}$ and HIC are greatly reduced, in comparison with the values obtained with the EPS helmet.

Table 3 of Kurt et al. gives $A_{max}$ for four helmets (EPS, and airbag at three pressures), each in two orientations, and (in most cases) using five drop heights. Kurt et al. noted bottoming out in four cases, and I have omitted these in obtaining the results below.

- Plots of $\ln(A_{max})$ vs. $\ln(h)$ are good straight lines.
- For the EPS helmet, the slope is 0.68 and 0.69 for parietal and vertex orientations.
- For the six cases of airbag helmet (three pressures, two orientations), the slope is between 0.56 and 0.85, averaging about 0.7.

Slopes of 0.5, 0.7, 0.9 would respectively correspond to exponent n being 1, 2.3, 9.

For most of the eight regressions, the confidence interval for the slope is quite wide. Consequently there is not much evidence one way or the other about whether it is some particular value, such as 0.5 or 0.6, which would respectively correspond to n being 1 or the Hertzian value of 1.5. And there is not much evidence one way or the other about whether the slope is the same in all eight cases.

## *14.5 Seidi et al. (2015)*

### 14.5.1 Summary of experiments

Seidi et al. (2015) apparently chiefly had in mind falls of the elderly when they conducted some experiments in which adult Hybrid III dummies (5th percentile female and 50th percentile male) fell from a standing position in five orientations on to a hard surface.

The surface was a force plate. The dummy's head had an accelerometer. From the measurements of force and acceleration, the effective mass of the head could be inferred.

For a given orientation of the dummy, there was no attempt to manipulate impact speed. Some variation in impact speed arose from the different orientations of fall.

I will treat two pairs of orientations as being equivalent, in the sense of attributing differences in maximum acceleration to differences in speed.

### 14.5.2 Results

Averages of several falls in each orientation were reported by Seidi et al. (2015) in their Tables 1 and 2.

I think impacts to the back of the head from falls in the no hips bend orientation, and from falls in the hips bend orientation, are probably similar.

- For the 5th percentile female dummy, the two orientations differed by a factor 1.26 in respect of impact speed, and a factor 1.27 in respect of maximum acceleration. This implies an exponent of 1.03 and therefore n = 1.06.
- For the 50th percentile male dummy, the two orientations differed by a factor 1.39 in respect of impact speed, and a factor 1.53 in respect of maximum acceleration. This implies an exponent of 1.28 and therefore n = 1.79.

I think impacts to the front of the head from falls in the no knees bend orientation, and from falls in the knees bend orientation, are probably similar.

- For the 5th percentile female dummy, the two orientations differed by a factor 1.32 in respect of impact speed, and a factor 1.31 in respect of maximum acceleration. This implies an exponent of 0.97 and therefore n = 0.94.
- For the 50th percentile male dummy, the two orientations were so similar in respect of impact speed that it is not sensible to attempt to estimate n.

## *14.6 Haddadin et al. (2009)*

Haddadin et al. (2009) reported results from a Hybrid III dummy being struck in the face by a robot, impact speeds being between 0.2 m/sec and 4.2 m/sec. As the impactor was aluminium and seems to have moved at constant speed during the impact (see next paragraph), it is appropriate to discuss the results in this chapter.

Some important aspects of the experiments are not clear to me, but I think the impactor on the robot moved at constant speed during the impact, and deformation and acceleration of the headform occurred. If the impactor did indeed move at constant speed, this is equivalent to the headform hitting a very massive object. (If deformation of the headform does indeed occur, the validity of the results relies on the headform being biofidelic in this respect. Haddadin et al. are of the opinion that the headform is appropriate for evaluating contact forces that are related to fractures of the human frontal bone.)

The following results refer to impacts at between 1.5 m/sec and 4.2 m/sec.

- Plotting ln(HIC) versus ln(v), the slope was 3.20 for robot KR6 and 2.90 for robot KR500. These correspond to exponent n being 2.7 and 1.7.
- Plotting ln($A_{max}$) versus ln(v), the slope was 1.50 for robot KR6 and 1.49 for robot KR500. These correspond to exponent n being 3.0 and 2.9.
- Plotting ln(HIC) versus ln($A_{max}$), the slope was 2.12 for robot KR6 and 1.94 for robot KR500. The first of these corresponds to exponent n being 4.2; the second is not compatible with the proportionality results of sections 4.5.1 and 6.5.5. (For a possible reason, see Appendix 5.)

Other papers relevant to robot impacts with humans are discussed in section 13.7.

## *14.7 Coats and Margulies (2008)*

Coats and Margulies (2008) reported results of head-first drops of an instrumented dummy that represented an infant 1.5 months old. Drop heights were 0.3 m, 0.6 m, and 0.9 m, and the impacted surface was a mattress, a carpet pad, or concrete. Coats and Margulies were concerned both with what can happen accidentally, and with what is unlikely to happen accidentally and thus is suggestive of abuse.

Considering the slope of the dependence of ln(maximum force) on ln(drop height), the possible range that is compatible with the results of section 4.5.1 is from 0 to 1. For the concrete surface, this slope is 1.2. However, the standard error of this is sufficiently large that the data can be said to be compatible with a slope in the given range.

## *14.8 Athletes' shoes and dogs' paws*

Cámara Tobalina et al. (2013) reported results from athletes dumping down on to a force platform. Averages of maximum force for three jumps of 13 participants were given for the combinations of two jump heights (30 cm and 60 cm) and two types of footwear. I have calculated that these imply exponent n was about 1.4 for basketball footwear and about 1.1 for running footwear.

Miao et al. (2017) report finite element modelling of dog paw pads striking rigid ground at speeds between 0.05 and 0.4 m/sec. They used two models, which they referred to as structured (structured epidermis, that is) and uniform, the latter being the simpler.

- In the case of the structured model, I find that the increase of ground reaction force with v suggests that exponent n is about 1.2, and the increase of S with v suggests that exponent n is about 1.4.
- In the case of the uniform model, exponent n calculated from both dependent variables is about 1.4.

## *14.9 Short duration of impact and intracranial pressure*

The paper by Pearce and Young (2014) was mentioned in section 3.5, and some of the data discussed in section 8.7. It is noted again at this point because of its argument that if impact duration is very short, intracranial pressure is much larger than would be expected on the basis of the maximum force. Impact duration is often short in the case of impact with rigid surfaces.

# Part C: Further areas of application

Chapters 15 and 16 will be on penetrating injury, and then chapters 17 and 18 will discuss impact tests relevant to damage of things. There is a little about data on penetrating damage in sections 16.5 and 16.6.

These chapters are generally similar in nature to chapters 7 - 14. There is some theory, and published results are examined in the light of theory. Three points in particular are worth making.

- In part, theory relates to one dependent variable and its relationship to one independent variable. Most interest is concentrated on two types of relationship. (See also especially sections 5.4 and 20.1.) The first type is a power function: the exponent is estimated from the data, and the exponent n that appears in the differential equation for force is then calculated. The second type is derived from n being a known value (such as 1, or perhaps 0 or 1.5 or 2), along with some other phenomenon.
- And in part, theory relates to consistency within a whole set of relationships between a number of dependent variables and a number of independent variables. For example, this means that the exponents of power functions need all to be consistent with some common value of n.

Part C has the following chapters.

15. Penetrating injury: Theory.
16. Penetrating injury: Data.
17. Damage to manufactured items.
18. Damage to fruits, vegetables, and eggs.

# 15. Penetrating injury: Theory

According to Breeze et al. (2013), physical models are essential in predicting the effects of ballistic injury and comparing potential protection measures. Ballistic injury here refers to penetrating injury from fragments, such as those from improvised explosive devices, and bullets. Physical models mostly refers to gelatin and animal flesh.

This chapter will consider some theory for the forces acting on a projectile (such as a bullet or a metal fragment) travelling through soft tissue. In testing, human tissue might be represented by gelatin (usually 20 per cent or 10 per cent) or by animal tissue. Theory here is in the form of differential equations. Proportionality relationships are obtained for S, depth of penetration; acceleration-based measures such as $A_{max}$ and HIC are not of interest. For residual velocity when a projectile passes completely through a target, see section 16.6.

Notation is as in chapter 4: m is mass of projectile, x is displacement, x' is the first differential (velocity), and x'' is the second differential (acceleration). The variable d usually represents projectile diameter, though it may be something else.

## *15.1 Theory 1*

Suppose that the following differential equation represents the deceleration of the projectile.

$m.x'' + k.((x')^2 + \beta).d^s = 0$

This equation only holds until velocity x' has fallen to 0. The other term also is then assumed to become 0, and the projectile remains stationary. The variable d (projectile diameter) is assumed to have the same proportionate effect on both terms. The exponent s is expected to be positive if d refers to projectile diameter.

If that equation holds, then depth of penetration S obeys the following proportionality relationship.

$S \propto d^{-s}.m.\ln(1 + \beta^{-1}.v^2)$

This was obtained as follows. The differential equation was input to wolframalpha.com, together with the initial conditions x(0) = 0 and x'(0) = v. The output is a fairly complicated expression. The expression is only valid up to the moment when the direction of travel would otherwise reverse, i.e., up to the time when x' has fallen to 0. Using that to identify depth of penetration, the result is the given equation.

The equations here are similar to equations (4) and (6) of Segletes (2008). A review by Ben-Dor et al. (2015) of penetration of concrete, to be briefly mentioned in section 16.5.2, may have some results relevant to this theory.

For this theory, and for the others below, S is proportional to a function of d (alone) multiplied by a function of m (alone) and by a function of v (alone).

## *15.2 Theory 2*

Suppose that the following differential equation represents the deceleration of the projectile.

$m.x'' + k.(x' + \beta).d^s = 0$

The exponent of x' is 1 instead of 2.

If this equation holds, the same method as in section 15.1 finds the following proportionality relationship.

$S \propto d^{-s}.m.(v - \beta.\ln(1 + \beta^{-1}.v))$

This has previously been given by de Bruyn and Walsh (2004).

## *15.3 Theory 3*

Suppose that the following differential equation represents the deceleration of the projectile.

$m.x'' + k.((x')^2 + \beta.x').d^s = 0$

This is similar to the model of Sturdivan (1978). Sturdivan describes the quadratic term as overcoming the inertia of the gelatin which must be moved aside as the projectile penetrates, and the linear term as viscous friction. The variable d (projectile diameter) is assumed to have the same proportionate effect on the linear and quadratic terms.

If the above equation holds, then from Sturdivan's equation (2), it may be found that depth of penetration S obeys the following proportionality relationship.

$S \propto d^{-s}.m.\ln(1 + \beta^{-1}.v)$

## 15.4 Theory 4

Suppose that the following differential equation represents the deceleration of the projectile.

$m.x'' + k.((x')^2 + \beta.(x')^q).d^s = 0$

The exponent q is between 0 and 1, and thus Theory 4 subsumes Theory 1 ($q = 0$) and Theory 3 ($q = 1$).

This model is similar to equation (14) of Segletes (2008). If the above equation holds, then according to Segletes' equation (17), depth of penetration S obeys the following proportionality relationship.

$S \propto d^{-s}.m.\ln(1 + \beta^{-1}.v^{2-q})$

## 15.5 Theory 5

Suppose that the following differential equation represents the deceleration of the projectile ($1 \leq q < 2$).

$m.x'' + k.(x')^q.d^s = 0$

If the above equation holds, then depth of penetration S obeys the following proportionality relationship.

$S \propto d^{-s}.m.v^{2-q}$

This is a rather simpler expression than the previous ones.

Theory 5 is perhaps the one most similar to the theories for headforms striking car exteriors or interiors (see especially chapters 4 and 5), as depth of penetration has power-function dependence on the independent variables. (However, m and v do not combine in the form $m.v^2$.)

The differential equation is not an entirely suitable assumption, as it implies velocity never reaches 0, and thus S here is the limiting distance beyond which the projectile does not pass, rather than the point at which velocity equals 0. This sort of objection, however, is usually regarded as an unimportant technicality. It may be said that a simple differential equation is only an approximation that is sufficiently accurate over a sufficiently wide range of velocity, that there are very likely other forces also, and thus the projectile does stop.

For residual velocity if a projectile passes completely through a target, see section 16.6.

According to Savvateev et al. (2001), the depth of penetration of a bullet into dry sand is found empirically to be proportional to $v^{0.4}$, provided v is less than a critical velocity at which the bullet melts.

## *15.6 Separate consideration of the initial contact preceding penetration*

In the context of penetration, most kinetic energy is typically lost in passage through the material, with deceleration according to equations such as those given earlier. However, there is extra energy absorption at the moment of initial impact, described by Sturdivan (1978) as backsplash and the generation of shock waves and surface waves.

Those initial effects occur even if the material is a homogeneous one such as gelatin. If the surface differs from the bulk of the material, there may be an additional effect: in the case of animal tissue, skin may be tougher than flesh, or in the case of granular material, the surface may be weaker because of looser packing.

## *15.7 Eisler et al. (2001)*

Theories 1 - 3 are special cases of force being a quadratic function of velocity: the linear term is missing in Theory 1, the quadratic term is missing in Theory 2, and the constant term is missing in Theory 3. The general case is discussed by Eisler et al. (2001, section 3).

- They note that usually both the linear and the constant terms are omitted.
- Eisler et al. consider that for small v, it may be realistic to omit both the quadratic and the linear terms, and they obtain a prediction that S is proportional to $v^2$. (For Theories 1 and 2, this is obtained by letting k tend to zero and $\beta$ tend to infinity while keeping the product $k.\beta$ constant.)
- For large v, Eisler et al. suggested that S is proportional to ln(v). Note, though, that Eisler et al. should not be interpreted as deriving the result from an assumption that force is proportional to the square of velocity: if the only force acting is proportional to the square of velocity, the projectile does not stop and S is not finite. (It may easily be checked that if $x(t) = \ln(1 + v.t)$, then $x(0) = 0$, $x'(0) = v$, and $x'' = -(x')^2$.)

Eisler et al. attempted to get an equation for small v, an equation for large v, and a transition between these. However, there may be extra energy absorption at the moment of initial impact. If that is so, there may be fairly simple dependence of S on an effective velocity, this being somewhat less than v, to reflect the extra energy absorption. A power function, as in section 15.5, might be fitted. An exponent of about 0.83, implying $q = 1.17$, appears appropriate for the data in Figure 14 of Eisler et al. (2001).

For impact with masonry rather than gelatin, the general quadratic case was discussed by Li Piani et al. (2017). They described the velocity squared term as inertial stress or quadratic drag, the linear velocity term as viscous resistance or Stokes' drag or shear resistance, and the constant term as related to the bearing or static strength of the target.

Li Piani et al. (2018) consider several formulae in which depth of penetration is proportional to impact velocity. In contrast, if one thinks that the dependence of force on instantaneous velocity x' is rather stronger than proportionality, one will probably also think that the dependence of depth of penetration on impact velocity will be weaker than proportionality. (Theory 5 is an example: if exponent q is greater than 1, exponent 2 - q is less than 1.)

A review by Ben-Dor et al. (2015) of penetration of concrete, to be briefly mentioned in section 16.5.2, may have some results relevant to this theory.

## *15.8 Relevance of Theories 1 and 2 to nonpenetrating (blunt) impacts*

It might seem that the literature of penetration injury, with its use of velocity-dependent forces suggested by theories of the drag on a solid moving through a fluid, would have nothing relevant to say about blunt impacts.

That is perhaps too pessimistic. Theories 1 and 2 have a constant term as well as a velocity-dependent term. It is possible they may be useful in blunt impact modelling, as competitors to the Hunt and Crossley equation.

- An impact surface might be designed to have approximately constant resistance (rather than being an approximately linear spring). Thus, having a spring exponent of 0 in Theories 1 and 2 might not be unrealistic.
- Energy absorption is often an important aspect of the total phenomenon; this might occur because of a velocity-dependent term, as in Theories 1 and 2.

In the equation in section 4.3, there are exponents n, p, and q, and a factor in powers of v, m, and d is present in the damping term. For comparison, in Theory 1, the exponents are 0, 0, and 2, and there is no such factor; and in Theory 2, the exponents are 0, 0, and 1, and again there is no such factor.

Proportionality relationships for maximum displacement have been given. Maximum acceleration occurs at first contact (i.e., when x' is at its greatest). It is proportional to $m^{-1}.(v^2+\beta).d^s$ (Theory 1) or $m^{-1}.(v+\beta).d^s$ (Theory 2).

## *15.9 Discussion*

Apart from the constant of proportionality, Theories 1, 2, 3, and 5 have one parameter for the dependence of S on v. However, a distinction might be made between the exponent q in Theory 5 and the β in the other theories: an exponent is likely to be considered constant and not dependent on the projectile or the material impacted, whereas β is a weighting factor between two mechanisms of deceleration, and is likely to be different for different projectiles and materials impacted.

- The more flexible theories might be preferred until and unless it becomes clear that a simplification is justified.
- However, there is typically quite a lot of scatter in empirical data (Segletes, 2008; Sturdivan, 1978), and adjusting β to the dataset may be using a degree of flexibility that is not justified by the consistency of the data. This is especially so because making allowance for effects at initial contact (see section 15.6) will require a parameter specific to the projectile and the material.
- Because of the scatter in the data, then, Theory 5 might be often chosen in the present state of knowledge.

Equations relevant to passage of a projectile through soft tissue may not be relevant to protection from such projectiles by textiles: there is discussion of mechanisms in Morye et al. (2000).

There is also considerable literature on penetration of a projectile into granular material. Examples are de Bruyn and Walsh (2004) and Savvateev et al. (2001), and see also sections 16.5.3 and 16.5.4. For a few words on penetration into concrete, see section 16.5.2.

# 16. Penetrating injury: Data

In this chapter there are four examples relevant to penetrating injury. In addition, there are a few words in sections 16.5 and 16.6 on penetrating damage (of, for example, concrete or clothing). It may be noted that penetrate is often used to mean that a projectile has entered a target, whereas perforate means that it has entered and exited.

## *16.1 Breeze et al. (2013)*

### 16.1.1 Introduction

Breeze et al. (2013) analysed experimental data on how far fragments penetrate into 20 per cent gelatin and into goat tissue. I have commented on that paper, and made some of the points below, in Hutchinson (2014d). In some contexts, depth of penetration is studied and the projectile does not exit. In other contexts, the projectile exits and its residual velocity is studied.

The fragments were of three sizes (differing in both diameter and mass); the largest was chisel-nosed. As is quite common, Breeze et al. took an entirely empirical approach, and made no use of theory.

Implications of Theory 5 (section 15.5) include the following. If force resisting the projectile is proportional to a power function of its instantaneous velocity, depth of penetration is proportional to a power function of impact velocity. The differential equation in section 15.5 is viewed as being a possible law of nature, with q correspondingly being a constant. Both $q = 2$ and $q = 1$ are thought to be realistic simple cases, respectively being "quadratic drag" and "linear drag" or viscous resistance. And if two values are considered plausible, it is reasonable to also permit intermediate values.

In contrast to the equation for S in section 15.5, Breeze et al. assumed depth of penetration is linear in v.

As mentioned in section 15.9, there is quite a lot of variability in empirical data (Breeze et al., 2013; Segletes, 2008; Sturdivan, 1978). Thus scatterplots do not clearly show what form the relationship takes. Choosing the best functional form is particularly difficult because phenomena not represented in the differential equation may occur at the initial impact, and the relationship between S/m and v may be changed as a consequence.

### 16.1.2 Impact velocity

Breeze et al. wrote their six straight-line relationships as S = intercept + slope.v. These can be re-expressed as $S \propto v - v_c$, where the critical or threshold velocity $v_c$ may depend on both what the projectile is and what the impacted material is; $v_c$ is the intercept on the velocity axis, and in terms of the original form of the equation is -intercept/slope. Ballistic limit is another name used, but some people may wish to reserve this term for $v_c$ as estimated by a particular method.

Thus in terms of Theory 5 (section 15.5), the linear relationships may be interpreted as exponent q being approximately 1, and v being replaced by a reduced speed $v - v_c$. As compared with simply accepting a linear relationship, an advantage of this viewpoint is that exponents are often viewed as having validity beyond the particular conditions of the experiment reported.

### 16.1.3 Projectile diameter

According to the equation in section 15.5, depth of penetration is proportional to projectile mass. Deviation from this when comparing the 0.16 gm and 0.49 gm projectiles is likely to be due to their different diameters.

If S/m is proportional to $d^{-s}.v^{2-q}$, where d is projectile diameter and s is another exponent, then the data for gelatin in Figures 5 and 6 of Breeze et al. may be used to estimate that s is approximately 2.1.

### 16.1.4 Projectile shape

If S/m is proportional to $d^{-2.1}$, the effect of the different shape of the 1.10 gm chisel-nosed projectile may be estimated.

- For impact with gelatin, impact speeds of 159 m/sec for the 0.49 gm cylindrical projectile and 190 m/sec for the 1.10 gm chisel-nosed projectile may be compared, as in each case these are 120 m/sec above the threshold velocity for penetration (39 m/sec and 70 m/sec).
- For the 0.49 gm and 1.10 gm projectiles, the respective observed penetrations were about 45 mm and 116 mm, a ratio of 2.58.
- If there is no effect of shape, this ratio would be predicted to be $(1.10/0.49).(5.4/4.0)^{-2.1}$, as the projectiles' diameters were 5.4 mm and 4.0 mm. That works out to be 1.19.
- The effect of the chisel nose thus appears to be to multiply penetration by about 2.2.

There is considerable uncertainty in the estimated shape effect of 2.2, as there are several steps to the reasoning. See also section 16.1.6.

### 16.1.5 Initial impact

Even with gelatin, there may be extra energy absorption at impact. Goat skin is sufficiently tough that the extra energy absorption is greater. Allowance is made by using an effective speed $v - v_c$. The correction $v_c$ is non-zero even for gelatin (and is as large as 70 m/sec in the case of the 1.10 gm fragment).

If depth of penetration is to be predicted using an effective speed, the question might be raised of how is this to be calculated.

- A reduction of speed by a certain amount independent of v was used in section 16.1.4.
- An alternative to a correction to speed is a correction to energy. That will lead to an effective speed being calculated as $\sqrt{(v^2 - v_c^2)}$. (If this reduction is because of penetration through something tough, it would probably depend on the material penetrated and the diameter of the projectile, but not on the projectile's mass or speed.)
- If this is reasonable, then the exponent $2 - q$ and the effective speed $v_e = \sqrt{(v^2 - v_c^2)}$ both create nonlinearity in the dependence of S on v. They could tend to cancel out, in which case the appearance of linearity would be misleading.

Residual velocity of a fragment penetrating through plywood is apparently usually modelled as proportional to $\sqrt{(v^2 - \text{constant})}$, rather than equal to it (Kaufman and Moss, 2015; Lambert and Jonas, 1976). See also section 16.6 below.

### 16.1.6 Reconsideration of the shape effect

In section 16.1.4, the effect of shape was estimated to be a factor of 2.2. This relied on some assumptions, one of which was that 159 m/sec for the 0.49 gm cylindrical projectile and 190 m/sec for the 1.10 gm chisel-nosed projectile are the same effective velocities. The following paragraphs consider what will be the effect of assuming the initial impact to reduce the projectile's energy by an amount that is independent of impact speed.

What impact speeds should be compared? For v = 159 m/sec for the 0.49 gm cylindrical projectile (threshold speed = 39 m/sec), the effective speed $v_e$ is $\sqrt{(159^2 - 39^2)}$, which is 154 m/sec. For the 1.10 gm chisel-nosed projectile (threshold speed = 70 m/sec), the equivalent impact speed v is 169 m/sec, as $\sqrt{(169^2 - 70^2)} = \sqrt{(159^2 - 39^2)}$.

Comparing 159 m/sec for the 0.49 gm cylindrical projectile with 169 m/sec for the 1.10 gm chisel-nosed projectile, the respective observed penetrations were about 45 mm and 95 mm, a ratio of 2.11. As earlier, this ratio would be predicted to be 1.19 if there were no effect of shape, and it is concluded that the effect of the chisel nose is to multiply penetration by about 1.8.

The decision about a reduction of speed or a reduction of energy seems to matter: the appropriate comparison speed was previously calculated as 190 m/sec, and is now 169 m/sec, leading to rather different estimates of the shape effect. Nevertheless, the conclusion is robust in a qualitative sense: the two estimates of a multiplying factor (2.2 or 1.8) are not very different, and the shape effect is of considerable magnitude. (Some other reasonable theory might, perhaps, lead to quite a different estimate, but it seems unlikely.)

Other examples of attempting to split an effect into two or more components are in sections 12.5.4 and 12.6.4.

### 16.1.7 Discussion

It has been possible to link theory about the forces acting on a projectile travelling through tissue with data on depth of penetration. This has allowed some specific additional conclusions to be drawn about the data of Breeze et al. It is noteworthy that the treatment of a relatively minor aspect of the data (threshold velocity for penetration of the surface) affects the interpretation of the major features.

Issues that were beyond the scope of the paper by Breeze et al. have not been discussed, including the possible tumbling and fragmentation of the projectile, and whether depth of penetration is more or less useful than other physical measures such as volume of cavity.

Concerning tumbling and fragmentation of the projectile, my impression is that the literature often does not deal with this very well. Suppose chief interest is in S observed in the non-tumbling condition, and that the tumbling observations are quite a small proportion of the total. (It is likely that S in the tumbling condition will be appreciably smaller than S in the non-tumbling condition.)

- If the non-tumbling observations are the ones of interest, it is desirable to include only them in the data analysis.
- It may not be possible to distinguish between tumbling and non-tumbling observations. If so, it would be desirable to devise some method of data analysis that would regard the observations as a mixture of two types (tumbling and non-tumbling), and would attempt to describe the majority (i.e., non-tumbling).

- Simply describing the mean of both conditions together would not properly represent the non-tumbling condition.

## *16.2 Jussila et al. (2005)*

I am inclined to think that separate study is desirable of the minimum speed to penetrate or perforate ($v_c$), and of the shape of the relationship at higher speeds. Without separate study, an opinion about $v_c$ is likely to distort perception of what the general shape of the relationship is, and an opinion about the shape of the relationship is likely to distort estimation of $v_c$. (By separate study, I mean determination of what independent variables affect $v_c$ or S, and what a suitable equation is.)

Another paper that fitted straight-line dependence of S on v was that of Jussila et al. (2005). Lead spheres were shot into gelatin blocks covered with skin simulant. (The intention was to evaluate various skin simulants for suitability.) Figure 2 of Jussila et al. shows depth of penetration versus impact speed, for simulant S7.

- The relationship appears to be approximately a straight line.
- For comparison, I have also plotted $S^2$ versus $v^2$. This also appears to be approximately a straight line. This would be the case if the energy lost at impact did not depend on v. (Other examples of two alternative relationships both being approximate straight lines occur in sections 18.8 and 18.12.)
- Using the original linear relationship between S and v, $v_c$ is estimated to be 91 m/sec. Using the linear relationship between $S^2$ and $v^2$, $v_c$ is estimated to be 100 m/sec. So for this dataset, it seems that the estimate of $v_c$ is fairly sensitive to what model is assumed. (See also the discussion in section 16.6 below of $v_c$ in experiments of Meyer et al., 2018.)

## *16.3 Krebsbach et al. (2012)*

The main topic of the paper by Krebsbach et al. (2012) was bacterial distribution along the wound track of a projectile. That is not a concern of this book, but comment can be made on some data in that paper. Krebsbach et al. fired projectiles into gelatin that was sufficiently thin that the projectiles exited. Table II of Krebsbach et al. gives the percentage of kinetic energy lost from entry to exit.

From the kinetic energies reported in Table II, the following may be calculated (Hutchinson, 2014b).

- As reductions in speed are all small (between 7 per cent and 16 per cent), the force acting on the projectile can be assumed to be approximately constant. Distance of travel through the gelatin was the same at all impact speeds. Thus force is proportional to change of energy.

- When average speed changed by a factor of 3.6, the corresponding change in force was a factor of 5.1. This summary was obtained from the average speed (i.e., the average of impact and exit speeds) changing from 51 m/s to 185 m/s, and the energy lost changing from 15 J to 77 J.

Theory 5 of section 15.5 predicts power-function dependence of force on speed. The exponent q of section 15.5 may be estimated as ln(5.1)/ln(3.6), which is 1.27. That is, instantaneous force is approximately proportional to the 1.27 power of instantaneous speed.

## *16.4 Anderson et al. (2016)*

Experiments on the penetration of arrows into gelatin were conducted by Anderson et al. (2016). They employed low speeds, between 8 m/sec and 14 m/sec. Anderson et al. were interested in one organism penetrating another (for example, in order to inject venom). Anderson et al. found that kinetic energy is a better predictor of depth of penetration than momentum or velocity.

Twenty data points (including m, v, and S) are given in the Supplement to the paper. I have taken logarithms and obtained regression results.

- Regressing ln(S) on ln(m) and ln(v), the coefficients are found to be 1.1 and 2.2.
- For the models of sections 15.1 - 15.5 of this book, S is proportional to m. That suggests regressing ln(S/m) on ln(v). The coefficient is found to be 2.0.

As noted earlier, the impact speeds in this experiment were much lower than usual. The projectile was also unusual. I do not know much about the topic, but I might make two points.

- The model should not be treated as a general finding that can be transferred to more the more usual conditions.
- But more specifically, the results could have implications for the concept of a minimum speed to penetrate, and what that speed might be.

## *16.5 Penetrating damage of things*

### 16.5.1 Introduction

Penetrating injury of humans (and penetration of gelatin, considered as a proxy for a human) is a step away from blunt injury. Blunt damage of things is also a step away, and will be discussed in chapters 17 and 18. Penetrating damage of things is two steps away from blunt injury, and will now be given some brief consideration. (As noted earlier, the word perforation is sometimes used to imply that a projectile has passed through a target; the word penetration does not carry this implication.)

My impression is that in modelling penetrating damage, a lot of attention is given to properties of the material that might be penetrated (e.g., concrete), to properties of the projectile, and to using these in predicting penetration.

### 16.5.2 Concrete

Forrestal et al. (2003) reported experiments on the penetration of concrete, and their equation (8) expresses maximum penetration in terms of impact speed. Approximately, S is proportional to $m.v^2$, with a correction term that reflects special phenomena occurring at initial contact. For some of the data in Forrestal et al. (2003) and Forrestal et al. (1996), I have plotted ln(S) vs. ln(v). The scatterplots are good straight lines, and the slopes are close to 2. The slopes would be 2 if an appropriate small constant were first subtracted from S, and the dependent variable were ln(S - constant).

Ben-Dor et al. (2015) have discussed a number of formulae relevant to the penetration of concrete. They concentrate on force being a quadratic function of instantaneous velocity (and special cases of this), except possibly during the early stages of penetration. Thus it may be that some of their results are relevant to the models in sections 15.1 and 15.7. See also Rahman et al. (2010).

### 16.5.3 Space

In space, the type of impact of greatest concern may be that of small objects travelling at very high speeds. Nishida et al. (2009) report experiments in which an impactor of mass approximately 11 gm and velocity approximately 1 km/sec struck an aluminium target. I find that a plot of ln(S) vs. ln(v) is a good straight line of slope 1.4. (That is for a target that did not have a thin aluminium or nickel plate attached.)

Another aspect of space exploration is the possible striking of a rocky or icy planetary body by a man-made object that will penetrate a few centimetres or perhaps metres. Some experiments on the penetration of ice at impact velocities of around 200 m/sec were reported by Suzuki et al. (2016). Their Figure 10 shows that penetration distance is approximately proportional to $v^2$. Alternatively, the relationship could be interpreted as linear beyond a threshold velocity necessary for penetration to occur. Hail is ice; for damage by hail, see section 17.8.

Mizutani et al. (1983) conducted experiments on the penetration of aluminium projectiles into sand. These experiments were intended to be relevant to the cratering of planetary bodies.

- Table 2 of Mizutani et al. refers to loose sand. I omitted three experiments having unique values of m and thus providing no information about the effect of v. There remained two experiments at one value of m, and four at another. Regressing ln(S) on ln(v), a slope of 0.30 is found.
- Table 3 of Mizutani et al. refers to compacted sand. I omitted four experiments having unique values of m, and three experiments for which v was not given. There remained 31 experiments (there were three different masses of projectile). A plot of ln(S) vs. ln(v) suggested straight line relationships, with all three having the same slope. That was estimated as 0.33.

### 16.5.4 Granular material

Table 5 of Kuutti et al. (2012) gives data on the penetration by canisters into LECA (granular lightweight expanded clay aggregate). The canisters in most cases were of mass 5 kg (or higher mass in a few tests), and the impact speeds were between 40 and 100 m/sec. I have plotted ln(S) vs. ln(v) for three sets of experimental conditions.

- I omitted two tests because the notes in Table 5 of Kuutti et al. indicate that these were unusual.
- I omitted one test because it was the only one in a certain set of conditions, and therefore could not give any information about the effect of impact speed. I omitted two tests because they were the only ones in a certain set of conditions and were very close in impact speed, and therefore could give almost no information about the effect of impact speed.
- There remained 21 tests in three sets of conditions. (However, the numbers of tests were unequal: 14, 3, and 4.)
- The plot of ln(S) vs. ln(v) is consistent with three straight lines of the same slope.
- Fitting a common slope (but permitting a different intercept for each set of conditions), this was found to be approximately 0.9.

This example is rather different from others, in that the penetrative damage of the LECA is not the centre of interest; rather, it is the potential for damage to the canister that is of concern, and the source of that is (probably) the maximum force.

For impact with granular materials, see also sections 9.1.5, 15.9, and 16.5.3.

## *16.6 Penetrating damage of things: Residual velocity*

Thinking of humans, depth of penetration is of obvious relevance to injury. Thinking of things --- perhaps some sort of wall or clothing --- residual velocity of a projectile if it passes through is of obvious relevance to human injury. Thus research is carried out that presumes that the impacting projectile will penetrate completely through the object struck, and studies the dependence of the residual velocity $v_r$ of the projectile on its initial impact velocity v.

It is found empirically that below some threshold speed, the projectile does not pass through the target. To predict the residual velocity at higher speeds, a simple hypothesis is that initial impact reduces the projectile's kinetic energy by an amount that is independent of impact speed. This is expressed in the equation below, in which $v_c$ needs to be estimated, and the equation is valid when v exceeds $v_c$.

$$v_r^2 = v^2 - v_c^2$$

A two-parameter generalisation is that $v_r^2$ is proportional to, not equal to, $v^2 - v_c^2$ (Lambert and Jonas, 1976). I think I am right to say that if Theory 5 holds (see section 15.5), residual velocity (after passing through a specified thickness of material) is proportional to impact velocity. Thus if initial contact reduces the projectile's kinetic energy by a constant amount, and then Theory 5 holds, then $v_r^2$ will be proportional to $v^2 - v_c^2$. (Other justifications for this relationship may also be given.)

A three-parameter generalisation is that, for some exponent p, $v_r^p$ is proportional to $v^p - v_c^p$ (Lambert and Jonas, 1976). Lambert and Jonas (1976, p. 13) refer to this suggestion of theirs as being "inescapably empirical", that is, without theoretical support.

The dataset of Meyer et al. (2018) refers to a woven glass/epoxy composite target and a steel projectile. (Concerning fibre-reinforced composites, see section 17.15.) Meyer et al. fit the three-parameter function of Lambert and Jonas to two subsets of the data, in total 24 shots. These subsets describe results for different positions of the impact relative to the weave of the target material.

- Meyer et al. estimated $v_c$ to be 188 m/sec and 167 m/sec for subsets A and B.

- I find that the estimates are appreciably lower if the one- or two-parameter functions are used instead. (See also section 16.2 above.)

The one-parameter function of Lambert and Jonas is based on a simple idea (initial energy absorption does not depend on v), and is simple to use; the two-parameter function can be supported by justifications, and fitting two parameters is popular in very many contexts; the three-parameter function does not have theoretical support, is complicated to use, and is difficult to interpret.

# 17. Damage to manufactured items

## *17.1 Introduction*

Chapters 17 and 18 are about damage, rather than human injury. About half of chapter 17 is about packaging --- sections 17.2 - 17.5 are largely theoretical, and sections 17.6 and 17.7 are largely about data. Then sections 17.8 - 17.18 are on a variety of subjects, as follows.

17.8: damage from hail or windborne debris,
17.9: frontal damage of cars in crash tests and accidents,
17.10: aircraft landing gear tests (that were published as long ago as 1931),
17.11: protection (of roads and other structures) against rock falls,
17.12: damage to undersea pipelines,
17.13: impact with a beam,
17.14: axial impacts with tubes,
17.15: fibre-reinforced composites,
17.16: metal honeycomb,
17.17: impact of bags containing liquid,
17.18: shear thickening fluid.

There will be a little further discussion of damage of manufactured items in section 19.4.6. In section 4.2.4, several studies of damage were mentioned in the context of assuming a power-function relationship between deformation and force, or of other relationships that would be consequences of that. For a few words about penetrating damage, see sections 16.5 and 16.6. Chapter 18 will also be about damage, but to fruits and other agricultural produce.

The motivation for the work of Mindlin (1945) was safe packaging of large vacuum tubes. He considered a packaged item in a container as one mass connected to another by a spring, and analysed the motion of the item for various types of spring. This is analogous to a headform hitting a spring, except that in this latter case the headform parts from the spring when it rebounds. Mindlin assumed maximum acceleration is the appropriate measure of outcome, rather than anything more complex such as HIC.

The contributions of Mindlin's paper include the following.

- Several possible equations for force-displacement relationships are considered. The first is a linear spring, and the others have one, two, or three parameters and are a little more complex.
- An alternative way of expressing such an equation is as acceleration in terms of time (i.e., the acceleration pulse). Mindlin obtains equations in some cases, and for others shows the shapes of pulses obtained by numerical integration.
- The results obtained for these models include equations for maximum acceleration and maximum displacement.

- There is a limited amount of consideration of other issues such as there being damping that is proportional to velocity.

Maximum acceleration and maximum force are the measures most used in the context of damage; for some discussion of the paper by Suhir (1997), see sections 3.1 and 3.8. The following approach is quite a common one.

- Damage either occurs or does not. That is, there are no grades of damage; the outcome is binary.
- The main determinant of whether damage occurs is $A_{max}$.
- There is a threshold of $A_{max}$ such that damage occurs if and only if $A_{max}$ exceeds this threshold.
- However, if change of velocity deltaV is below some threshold, damage does not occur even if $A_{max}$ is very large.
- When a graph is drawn of this, it is quite common for the damage boundary to be shown as curved where $A_{max}$ and deltaV are both close to their respective thresholds.

## *17.2 Introduction to the cushion curve and the damage boundary curve*

### 17.2.1 Data presentation: Graphs with three independent variables

Two forms of data presentation that are used in packaging will be described below. These are the cushion (or cushioning) curve and the damage boundary curve.

Among the most important characteristics of a graph are the four following.

- What the horizontal axis is.
- What the vertical axis is.
- If several curves are shown on one graph, what variable varies between these different curves?
- If there is a set of several of these graphs, what variable varies between these different graphs?

### 17.2.2 The variables shown in a cushion curve

The phrase "cushion curve" is used in more than one sense. Frequently it is used as follows. This curve inter-relates four variables, the answers to the above questions being as below.

- Horizontal axis. The ratio of mass of impactor m to area, this ratio being referred to as static stress.
- Vertical axis. $A_{max}$.

- What variable varies between the different curves? Impact speed v (or alternatively drop height h).
- What variable varies between the different graphs? The cushioning material. Thus stiffness k and other material properties vary.

See also sections 17.3 and 17.4.

### 17.2.3 The variables shown in a damage boundary curve

The phrase "damage boundary curve" is used in more than one sense. Frequently it is used as follows. Maximum acceleration $A_{max}$ may be regarded as the fragility of an object: the acceleration at which it is damaged. The damage boundary curve shows, for a specified value of $\sqrt{(k/m)}$ (on the horizontal axis) and a specified value of $A_{max}$ (referring to a curve), at what speed the object is damaged. For the damage boundary curve, the information is organised as follows.

- Horizontal axis. $\sqrt{(k/m)}$, sometimes called a frequency parameter.
- Vertical axis. Impact speed v (or drop height h).
- What variable varies between the different curves? $A_{max}$.
- What variable varies between the different graphs? The damping parameter.

See also section 17.5.

## *17.3 Cushion curves*

### 17.3.1 Burgess (1990, 1994)

Packaging engineers make use of the "cushion curve". This is a plot of the outcome of an impact (usually maximum acceleration) versus static stress (which is often on a logarithmic scale). It usually refers to results of testing, with different curves referring to different drop heights of an impactor corresponding to the relevant static stress. The curve is U-shaped because of bottoming out (Wyskida and McDaniel, 1980, especially p. 103). Static stress is analogous to headform mass, and drop height is analogous to speed.

When foam or something else is being crushed between two stiff surfaces, bottoming out may be of a qualitatively different form from that occurring when a car bonnet bends until it suddenly comes into contact with a stiff structure (which is the form that is important in chapter 2).

Consider an object dropped on to a cushion. Suppose the mass of the object is m, its contact area with the cushion is *area*, drop height is h, and cushion thickness is *thickness*. If I understand Burgess (1990, 1994) correctly, he suggests that a set of cushion curves may all be summarised in the following way.

$A_{max}$.m/*area* = some function of ((m/*area*).(h/*thickness*))

A less ambitious hypothesis is that in the relevant experiment, this equation is followed for some constants *area* and *thickness*, but that these are not necessarily the contact area with the cushion and the thickness of the cushion.

Cushion curves are U-shaped because of bottoming out. It seems unlikely to me that such a simple equation in four variables would be valid in these circumstances. Partly because of that, and partly because they were not considered in the context of human injury, I will now omit *area* and *thickness*; and as h is proportional to the square of impact speed, I will replace it with $v^2$. The equation is closely related to the discussion of human injury in earlier chapters, as it may be written as follows.

$A_{max}$.m = some function of (m.$v^2$)

Thus $A_{max} \propto m^{-1}.f(m.v^2)$. If $f(m.v^2)$ is $(m.v^2)^{n/(n+1)}$, the proportionality relationship for $A_{max}$ given in section 4.5 is obtained.

## 17.3.2 Soper and Dove (1962)

Earlier, Soper and Dove (1962) had obtained a related expression by use of dimensional analysis.

For systems with a particular cushioning material, their equation (5) shows that 2.$A_{max}$.*thickness*/$v^2$ is a function of two variables, m.$v^2$/(2.*area.thickness*) and v/*thickness*. Consequently, $A_{max}$.m/*area* will be m.$v^2$/(*area.thickness*) multiplied by some function of m.$v^2$/(2.*area.thickness*) and v/*thickness*. Evidently the following is also true.

$A_{max}$.m/*area* = some function of m.$v^2$/(2.*area.thickness*) and v/*thickness*

As $v^2$ is proportional to h, Burgess' expression is a special case of this in which v/*thickness* has been omitted.

If this reasoning is correct, it would seem that if Burgess' expression were found to fail for some particular dataset, a sensible next step would be to examine whether the expression holds provided v/*thickness* is kept constant. Woolam (1968) reports such data in his Figure 11.

Barkan and Sirkin (1963) agree with Soper and Dove that one of the important quantities is m.$v^2$/(2.*area.thickness*). But in place of v/*thickness*, they propose m.*thickness*/*area*. My present opinion, however, is that the two ideas are equivalent.

- Notice that m.*thickness*/*area* = (*thickness*/v)$^2$ × 2 × m.$v^2$/(2.*area.thickness*).

- A function of $m.v^2/(2.area.thickness)$ and $m.thickness/area$ can therefore alternatively be seen as a function of $m.v^2/(2.area.thickness)$ and $v/thickness$.

Section 17.4.4 will examine data of Park et al. (2016) in the light of the work of Soper and Dave (1962) and Barkan and Sirkin (1963).

## *17.4 Modelling of bottoming out using cushion curves*

### 17.4.1 Model

The following is a possible model for bottoming out. It is a gradual or smooth model for bottoming out, and it may be realistic when foam is crushing more and more. (Presumably when the underside of a car bonnet strikes the engine, there is some sort of sudden, not gradual, bottoming out.) For references, see the final paragraph of section 17.4.2.

Consider a mass m, velocity v, striking a cushion.

- Suppose that force is proportional to $1/(t - x)$, where t is a constant and x is instantaneous deformation.
- Integrate force from $x = 0$ to $x = S$, the maximum deformation (displacement) of the cushion. Set this energy equal to the kinetic energy at impact.
- Elementary manipulations then give the following result. Maximum acceleration is $B_1.m^{-1}.\exp(B_2.m.v^2)$. The quantities $B_1$ and $B_2$ are likely to be affected by stiffness and thickness of the cushion.
- For applications of this expression to impacts on materials used as the liner of helmets, see sections 7.10.3 and 8.4.4.
- Considered as a function of m, the above expression is U-shaped.
- The same is true of stiffness: m ought to be interpreted as the ratio of m to k, the constant of proportionality between force and $1/(t - x)$.

The main point of this model is to include bottoming out (deformation less than t, no matter how large the force). The great advantage of assuming that force is proportional to $1/(t - x)$ is that the mathematical consequences are elementary.

- In respect of their detailed shape, cushion curves could be badly wrong. They are based on force being proportional to $1/(t - x)$. Firstly, for small x, this is not proportional to x (which is the usual assumption). Secondly, large x (that is, only a little smaller than t) corresponds to bottoming out; as far as I know, very little is known about force as a function of x when this happens.
- The usual graph of compressive stress versus strain shows proportionality at low stress and strain, a plateau where stress only gradually increases, and a sharp increase of stress at high strain, as the foam densifies. Approximating this by $1/(t - x)$ is in effect giving

attention to the plateau followed by the sharp increase, and the proportionality region at low stress and strain is ignored.

- If this model were found to be useful for relatively high energies of impact (approaching bottoming out), it would be expected that for many materials the model would fail at low stress and strain.
- It might perhaps be possible to work on the assumption that force is $k_2/(t - x)$, but with the exception that it is $k_1.x$ for small values of x.

### 17.4.2 Stress-energy method of constructing a cushion curve

If force is proportional to 1/(t - x), maximum deformation cannot exceed t. This expression may be regarded as a simple way of describing bottoming out. However, in packaging engineering, some further explanation and justification of the above is given.

- This is termed the stress-energy method of constructing a cushion curve.
- A cushion curve includes in the equation the thickness of the cushion and the area that is impacted; t is interpreted as the thickness of the cushion that is being deformed.
- Justification for the reciprocal of (t - x) lies in the equation for gases, Pressure × Volume = Constant. Volume is proportional to (t - x), and force is pressure multiplied by area. (That equation for gases assumes temperature is constant.)
- The above derivation and the resulting expression for maximum acceleration are regarded as especially suitable for closed-cell foams. In this case, force exerted might be largely due to gas pressure.

An expression for maximum acceleration was given in 17.4.1.

The following papers in particular were useful to me in writing sections 17.4.1 and 17.4.2: Burgess (1988, 1990, 1994) (though Burgess, 1988, seems to consider this model too simple), Daum (2006), and Ruiz-Herrero et al. (2005, 2006). There is a history to this topic dating from the 1960's, as Burgess (1988), in particular, makes clear.

### 17.4.3 Navarro-Javierre et al. (2012)

Navarro-Javierre et al. (2012) compared some methods of constructing cushion curves. In the course of this, they gave a graph of stress versus deflection for an expanded polystyrene and a polyethylene foam.

- *Expanded polystyrene, 50 mm thickness*. Increase of stress from 105 to 250 (kPa) corresponded to increase of deflection from 10 mm to 40 mm, and thus to a decrease of cushion thickness from 40 mm to 10 mm. Thus a factor of 2.4 results from a factor of 4.0. The exponent of t - x is about -0.6, rather than -1.0.

- *Polyethylene foam, 50 mm thickness*. Increase of stress from 45 to 225 (kPa) corresponded to increase of deflection from 10 mm to 40 mm, and thus to a decrease of cushion thickness from 40 mm to 10 mm. Thus a factor of 5.0 results from a factor of 4.0. The exponent of t - x is about -1.2, not very different from -1.0.

### 17.4.4 Park et al. (2016)

Park et al. (2016) report drop tests on to four materials (two types of corrugated paperboard, each with two types of flute). They had two independent variables describing the impact (drop height and mass), and two independent variables describing the cushion (area and thickness). The output variable was maximum acceleration.

Park et al. were aware of the work of Burgess that is described in sections 17.4.1 and 17.4.2 above. Consequently, for each of their four materials they fitted an equation representing a linear relationship between ln($A_{max}$.m/*area*) and (m/*area*).(h/*thickness*) (where h is drop height).

Using dimensional analysis, Soper and Dove (1962) obtained certain results that were discussed in section 17.3.2. Because of those results, I have fitted a regression equation for ln($A_{max}$.m/*area*) in terms of m.h/(2.*area.thickness*) and v/*thickness*.

- With m.h/(2.*area.thickness*) in the equation as a predictor already, v/*thickness* had a negative effect.
- That was true for each of the four materials considered separately. The effect of v/*thickness* was statistically significant in three cases (and almost so in the fourth case).

As far as I know, there is no reason to expect the effect of v/*thickness* to be linear and combine additively with the effect of m.h/(2.*area.thickness*) when predicting ln($A_{max}$.m/*area*); that form of equation has been used for simplicity and convenience.

In view of the proposal of Barkan and Sirkin (1963) that was discussed in section 17.3.2, I have fitted a regression equation for ln($A_{max}$.m/*area*) in terms of m.h/(2.*area.thickness*) and m.*thickness*/*area*.

- With m.h/(2.*area.thickness*) in the equation as a predictor already, m.*thickness*/*area* had a positive effect.
- That was true for each of the four materials considered separately. The effect of v/*thickness* was statistically significant in all four cases.

As far as I know, there is no reason to expect the effect of m.*thickness*/*area* to be linear and combine additively with the effect of m.h/(2.*area.thickness*) when predicting ln($A_{max}$.m/*area*); that form of equation has been used for simplicity and convenience.

According to the reasoning that I give in section 17.3.2, the two variables v/*thickness* and m.*thickness*/*area* are approximately equivalent as additional variables, once m.h/(2.*area.thickness*) is in the equation as a predictor. Consequently, I am not surprised at the similarities between the two sets of results.

Findings of effects of additional variables show the positive side of guidance from theory. I should perhaps note that there is potentially a negative side too, in that one may fail to search a dataset for additional features. And as mentioned in section 17.4.1, there are several rather unsatisfactory aspects of assuming force is proportional to 1/(t - x).

### 17.4.5 Relationship of $A_{max}$ to four variables

It was found in section 4.5.1 that $A_{max}.m \propto (m.v^2)^{n/(n+1)}$. It appears to me that the conclusions of Soper and Dove (1962) given in section 17.3.2 suggest how the geometric factors of area and thickness might be incorporated in this. Their expression takes this form if the two terms are combined as follows.

$A_{max}.m/\mathit{area} \propto (m.v^2/(2.\mathit{area}.\mathit{thickness}))^{n/(n+1)}.(v/\mathit{thickness})^0$

For a dataset in which *area* and *thickness* are available, as well as m and v, $\ln(A_{max}.m/\mathit{area})$ could be plotted versus $\ln(m.v^2/(2.\mathit{area}.\mathit{thickness}))$. If the relationship is approximately a straight line, the slope is interpreted as n/(n + 1).

## *17.5 Damage boundary curve*

As explained in section 17.2.3, a damage boundary curve shows v versus $\sqrt{(k/m)}$ for constant $A_{max}$. $A_{max}$ is the acceleration at which the object in question is damaged.

### 17.5.1 References for cushions defined by certain functional forms

Wang (2002a, b) has considered some particular forms of nonlinear cushioning.

- Zero damping, linear and hyperbolic tangent cushions. For these two cases, expressions for $A_{max}$ are given in Wang (2002a).
- Zero damping, cubic and tangent cushions. For these two cases, expressions for $A_{max}$ are given in Wang (2002b).

### 17.5.2 References for cushioning of a "key component"

There is sometimes interest in a system in which a manufactured item is cushioned from the environment, and a part of the item (referred to as a key component) is cushioned from the main part.

- Linear cushions: Wang and Jiang (2010).
- Cubic cushion (force equals $k_3.x^3 + k_1.x$): Jiang and Wang (2012).
- Tangent cushion (force approaches infinity as deformation approaches thickness, and force is proportional to tan((π/2).(deformation/thickness)): Wang and Jiang (2010)
- Hyperbolic tangent cushion (force approaches a limiting force as deformation approaches infinity): Jiang and Wang (2012).

### 17.5.3 Jiang and Wang (2012)

Again with a "key component" model, Jiang and Wang (2012) reported an experiment with an object constructed to have a cubic cushion. That is, a steel disk was connected by a (linear) spring to another, which was supported on an expanded polyethylene pad (having the characteristics approximately of a cubic spring) on a vibration platform.

Table 5 of Jiang and Wang gives $A_{max}$ observed at various values of deltaV (change of velocity). If ln($A_{max}$) is plotted versus ln(deltaV), the relationship is a very tight one. However, it appears not to be a straight line, but to have a slope of about 1.2 at the lower speeds and 2.1 at the higher speeds.

Table 6 of Jiang and Wang gives $A_{max}$ observed at six combinations of values of change of velocity and stiffness of the spring. Fitting a regression line, ln($A_{max}$) as a linear function of ln(deltaV) and ln(stiffness), the exponent of deltaV was estimated to be 1.1 and the exponent of stiffness was estimated to be 0.6. These would be 1.0 and 0.5 for a linear spring, and so it seems the behaviour of the intentionally nonlinear device is not far from linear.

## *17.6 Burkhard (1966)*

Figure 7 of Burkhard (1966) is a plot of maximum acceleration versus drop height for a hearing aid. (The experimental method used an impact from a pendulum covered with the relevant surface material, rather than a simple drop.)

- Separate results are shown for four floor surfaces.
- Logarithmic scales are used for both axes.
- The surfaces differed by a factor of about 15 in maximum acceleration.

The slopes (logarithmic scales) found were not very different for the four surfaces. They were approximately 0.85 (the hardest surface, bare metal), 0.65, 0.75, and 0.95 (the softest surface, carpet and foam pad).

If the exponent n were 1, maximum acceleration would be proportional to impact speed (see Table 4.2), and thus to the square root of drop height. The slope, when using logarithmic scales, would be 0.5. As the slopes are greater than this, the values of n must be greater than 1.

That n exceeds 1 agrees with the stiffness of transducer mounting increasing with compression and force, which according to Burkhard (p.172) is typical.

Zhou et al. (2008) used a pendulum impact method to mimic what might happen when a portable electronic device is dropped. Zhou et al. found that peak force was proportional to $v^{1.14}$ (v = impact speed). They were aware of Hertz contact theory (see section 4.2.4) and that it implies an exponent of 1.2 for this relationship.

## *17.7 Dropping boxes or packages*

Malasri et al. (2012) were concerned with damage from dropping plastic totes containing a variety of healthcare products. (A tote is a box.) In their Table 1, they report maximum accelerations of a product inside a plastic tote when dropped from various heights. The tote had one of three cushioning materials underneath the product, or no cushion.

In two of the conditions (no cushion, and the poorest cushion) a doubling of drop height led to approximate doubling of $A_{max}$. A doubling of drop height implies impact speed is multiplied by 1.41 (that is, $\sqrt{2}$). Multiplying impact speed by 1.41 implies that if exponent n is 1 (as it is for a linear spring), $A_{max}$ will be multiplied by 1.41 also, not by 2. If $\ln(A_{max})$ is plotted versus ln(drop height), the four relationships are seen to be approximately straight lines.

- In the two cases for which a doubling of drop height led to an approximate doubling of $A_{max}$, the slopes are 0.98 (no cushion) and 0.95 (poorest cushion).
- In the other two cases, the slopes are 0.84 and 0.69.
- These four slopes imply that the values of the exponent n are approximately 41, 20, 5, and 2.2.

Mattar Neto et al. (2008) conducted finite element simulations of a package impacting a rigid surface in three orientations (vertical, horizontal, corner). They reported both maximum acceleration and deformation. These were strongly affected (in opposite directions) by orientation. The relationships in section 4.5.1 imply thar $A_{max}$ and S should be inversely proportional if stiffness k changes, and this will be so whatever exponent n is. For the three pairs of observations reported by Mattar Neto et al., the estimated exponent is -0.96. Change of orientation has complicated effects on acceleration and deformation, but evidently these are roughly equivalent to a change of stiffness.

For dropping a box, see also section 18.6.

## *17.8 Damage from hail and windborne debris*

Rain and hail can cause damage to manufactured items, structures, plants, soil, and other things. The same is true for objects blown by the wind.

The three papers discussed here have several authors in common. I should mention that the important part of the impacts (maximum force, and close to it) was very brief, with correspondingly high forces, and it occurred very early in the total impact. To some extent, it was more similar to the common engineering example of steel ball striking steel plate than are most of the examples in this book. See chapter 14 for impact between a human and a rigid surface, which has similarities.

Perera et al. (2018) reported peak contact force for manufactured hail ice (mass 75 gm), either spherical or irregular, at various velocities between 25 m/sec and 45 m/sec. Plotting ln(maximum force) versus ln(v), I find the relationship is a good straight line. (There is hardly any difference between the results for spherical and irregular ice.) The slope is 1.44, corresponding to exponent n being about 2.6.

Sun et al. (2015) reported peak contact force for spheres of hail ice (mass between 60 and 130 gm), at various velocities between 10 and 30 m/sec. The following refers to data in Figure 22 of Sun et al.

- *Sphere of mass 59.5 gm (diameter 50 mm)*. I have compared the logarithm of the ratio of the corresponding forces with the logarithm of the ratio of the highest and lowest impact energies. The ratio of the logarithms was 0.57. That means exponent n is about 1.34, not far from the Hertzian value of 1.5.
- *Impact velocity 16.8 m/sec*. Examining the effect of mass at a given velocity is more difficult, as sphere diameter changes also. As mentioned in section 4.3.2, for Hertzian impact, exponent n is 1.5 and the exponent (s) of the impactor diameter is 0.5. Thus the prediction from section 4.5.1 is that maximum force is proportional to diameter$^{0.2}$.m$^{0.6}$.v$^{1.2}$. The two velocities are the same, and so what matters is diameter$^{0.2}$.m$^{0.6}$. For spheres of masses 59.5 gm and 131 gm (diameters of 50 mm and 62.5 mm), the ratio of maximum force is consequently predicted to be 1.7. In Figure 22 of Sun et al., it appears to be about 1.5.

Perera et al. (2016) reported peak contact force for spheres of four materials (mass between 90 and 300 gm), at various velocities between 10 and 40 m/sec. For each material, I have compared the logarithm of the ratio of the corresponding forces with the logarithm of the ratio of the highest and lowest velocities. In each case, the ratio of the logarithms was not very different from 1 (the range was from 0.8 to 1.1). Hertzian impact theory would imply a value of 1.2. On impact, the wooden sphere rebounded, but the others (brick, and two types of concrete) disintegrated. Even so, Perera et al. may consider Hertzian impact theory to be relevant, as they note that fracturing occurs after peak force is reached.

The three papers made some some use of the Hunt and Crossley equation. (See sections 4.2.5 and 4.3 for this.) However, this use seems to have been in connexion with the force-time and force-displacement relationships, rather than the implications for the dependence of maximum force on impact velocity. In addition, Sun et al. and Perera et al. permitted the exponent to be different for different impact velocities.

For penetration into a planetary body made of ice, see section 16.5.3.

## *17.9 Car crashes*

Crash tests are often conducted in which a car strikes a solid block. Among the quantities of interest is the acceleration pulse of the undamaged part of the vehicle. Important characteristics of the pulse include maximum acceleration, duration of impact, and vehicle deformation. Much crash test data exists, but most is at the few speeds specified by particular test protocols. There is rather little at other speeds.

Hutchinson (2016) discussed four papers on crash tests, and one on data from collision recorders operating in real accidents (Gockowiak et al., 2014; Neilson et al., 1968; Wall et al., 1970; Woolley, 2001; Wood et al., 2003). Maximum deformation S was the chief dependent variable.

- In the test data, there were at least four data points for three models of car. A plot of deformation vs. speed on logarithmic axes appeared to be a straight line in each case.
- The exponent was approximately 1.37, and thus the exponent n was approximately 0.46, for three models of front wheel drive cars.
- For the real crash data, n was between 0.32 and 0.71 for twelve models of car.

It might tentatively be suggested that if better information is not available, n might be taken as 0.5 for front wheel drive cars.

Han (2007) discussed some results from low-speed front-to-rear impact tests. Two of these (referred to as Crashes 9 and 10) were very similar except for speed, that is, a particular model of moving vehicle struck a particular model of stationary vehicle. The speeds were 3.5 km/h and 7.5 km/h, i.e., they differed by a factor of 2.5. As I understand the results in Table 2 of Han (2007), maximum deformation changed by a factor of 2.2. That suggests exponent n is about 1.3.

> Results for $A_{max}$ in Han (2007), in Han (2016), and in Hutchinson (2016) are also suggestive of values above 1. My opinion at present is that there is a danger that $A_{max}$ might be misleading, as it refers to a moment rather than to a substantial fraction of the impact. (On the other hand, $A_{max}$ often has obvious direct appeal, as it refers to an object that is representing a human or which is sustaining the damage of interest.)

Quite a lot of people are interested in crash reconstruction, and they may make use of deformation vs. speed relationships found in crash tests. Unsurprisingly, there are many difficulties in crash reconstruction. One of them is that very few high speed crash tests have been conducted. For example, in routine crash test data used by Han and Kang (2016), nearly all the crashes were at 40 km/h or 56 km/h, and there were none at higher speeds. Han and Kang also considered eight tests from the Korea Automobile Testing and Research Institute; among these there was one at 64 km/h and one at 79 km/h, but neither was a full frontal impact.

For rigid pole impact tests of three models of car, Craig (1993) examines data at five impact speeds between 10 mile/h and 45 mile/h. (Thus the ratio of the highest to the lowest impact speed was 4.5.)

> Plotting ln(crush distance) vs. ln(v), with a common slope for the three models of car but different intercepts, I find a slope of 0.76. Permanent deformation is probably not very different from maximum deformation. Thus if the proportionality relationships in section 4.5.1 apply to this type of impact, exponent n is about 1.6.

An important use for such data in in crash reconstruction, to estimate impact speed from the amount of vehicle damage. Making the calculation that way round (i.e., speed from crush distance), Craig found v was approximately proportional to crush distance, and that the prediction was improved if a small amount that increased with crush was added. That is consistent with the exponent of v (when calculating crush distance from speed) being less than 1.

For deformation of tubes subjected to axial impact, see section 17.14. Deformation of a ship's bow on impact with a rigid wall was briefly discussed in section 4.2.4.

## *17.10 Aircraft landing gear*

Peck and Board (1931) reported results of drop tests (from up to 24 inches) of a light aircraft fitted with one of three types of landing gear.

Their Figure 14 shows both compression S and maximum force, plotted versus drop height. In addition to the empirical data, curves are shown.

- Compression is shown as a curve through the origin (0, 0) that is concave downwards.
- Maximum force is shown as a curve through the origin (0, 0) that is concave downwards.

I have estimated exponents in the usual way. They are about 0.25 for compression, and 0.50 for maximum force. If compression and maximum force can be interpreted in the same way as in section 4.5, these values are not compatible. That for compression suggests that exponent n is about 3, whereas that for maximum force suggests n is about 1.

Looking at compression and maximum force versus drop height in Figure 14 of Peck and Board (1931), curvature of the relationship is not obvious in either case. Curvature becomes apparent only when one insists that the dependent variable must be 0 when drop height is 0.

> Now an advantage is seen of having a theory that identifies the two relationships as incompatible, namely, that it provides a reason for questioning whether both dependent variables really must be predicted to be zero when drop height is zero.

It has been noted at several points in this book (e.g., sections 5.4 and 6.4) that it may be fairly simple to model why predictions of section 4.5 apparently fail (some special phenomenon at initial contact, for example).

## *17.11 Rockfalls*

### 17.11.1 Yuan et al. (2015)

A structure to protect buildings or roads from rockfall can be considered a manufactured item and included in this chapter.

Yuan et al. (2015) conducted experiments in which rocks fell on to a steel plate with a cushion layer of gravel, sand, or clay. Rock mass and speed varied, and were approximately 5 kg and 4 m/sec. A transducer measured the impact force.

Yuan et al. (their equation 4) found that impact force is proportional to $m^{0.22}.v^{0.50}$.

Measuring impact force with a transducer may be sufficiently similar to measuring maximum force with a headform for the relationships in section 4.5 to be relevant. Multiplying the proportionality relationship for $A_{max}$ by m in order to get maximum force, the result is $m^{n/(n+1)}.v^{2n/(n+1)}$ (see section 4.5.2). For the empirical relationship found by Yuan et al., the exponent of v is approximately twice the exponent of m, and thus is consistent with the suggested form of relationship; n is approximately 0.32.

### 17.11.2 Ho and Masuya (2013a)

Ho and Masuya (2013a) report a study using the finite element method. The model was of a sphere (mass 260 kg) falling on to a sand cell. Condition FD refers to the sand cell having four lateral faces free to deform; condition MC refers to material confinement, that is, the sand cell being surrounded by sand. Drop height h varied from 3 m to 10 m.

From the results given, it may be found that depth of penetration S was approximately proportional to $h^{0.36}$ in the FD condition, implying that n = 1.79, and to $h^{0.53}$ in the MC condition, implying that n = 0.90.

In Figure 18 of Ho and Masuya (2013a), it appears that when h is multiplied by 3.33, maximum impact force is multiplied by about 1.8 (in both conditions). This would imply that n is about 0.95.

### 17.11.3 Ho and Masuya (2013b)

Ho and Masuya (2013b) dropped an impactor (mass 7.2 kg) on to a bed of sand or gravel. There was an accelerometer inside the impactor. Depth of penetration S was measured by double integration of acceleration. Drop height h varied from 0.5 m to 2 m.

From the results given, it may be found that for sand, S was approximately proportional to $h^{0.69}$, implying that n = 0.46. For gravel, S was approximately proportional to $h^{0.57}$, implying that n = 0.75.

Ho and Masuya also note that there is a design formula for impact force due to a rock fall. This expression is proportional to $m^{2/3}.h^{3/5}$. The theory in section 4.5.1 of this book would suggest $m.A_{max}$, which is proportional to $m^{n/(n+1)}.h^{n/(n+1)}$. In the expression reported by Ho and Masuya, the exponents of m and h are similar but not exactly the same (0.67 and 0.6). It is interesting, though, that these values would suggest n is 2 or 1.5, rather different from those found from the variation of depth of penetration with drop height (0.46 for sand, and 0.75 for gravel).

### 17.11.4 Zhu, Wang, et al. (2018)

Zhu, Wang, et al. (2018) conducted experiments in which spheres of sizes up to 5 cm radius fell from heights of up to 1.2 m on to granular cushions of various thicknesses and particle sizes. Damage depth was reported in their Table 5, and so was coefficient of restitution. In 32 test conditions, coefficient of restitution did not vary very much (the range was from 0.21 to 0.36). The following results concern damage depth. (It seems likely there was deformation of the striking spheres as well as of the struck cushions.)

- Zhu, Wang, et al. did not use theory, but fitted a simple additive model of four factors.
- I fitted a model in which ln(damage depth) was linearly dependent on ln(movement height), with the intercept being a simple additive model of the remaining three factors.
- The slope (that is, the coefficient of ln(movement height)) was estimated to be about 0.5. If it is appropriate to interpret movement height as proportional to conventional drop height and thus to the square root of impact speed, and to interpret the slope in terms of exponent n, then n is about 1.

## *17.12 Impact with undersea pipelines*

Undersea pipelines may be struck by anchors, trawl gear, or other things. Zhu, Liu, et al. (2018) dropped a wedge-shaped indenter, loaded to various masses and at various speeds, on to pipes. Data obtained include maximum force (maximum acceleration was measured and multiplied by mass) and five measures of deformation.

I have conducted some elementary analyses of the data for pipe of type B. There were 16 impacts; mass varied by a factor of about 1.8, and impact speed varied by a factor of about 3.4.

- After taking logarithms, relationships with impact speed were good straight lines at each mass.
- Zhu, Liu, et al. used impact energy ($\frac{1}{2}.m.v^2$) as an important predictor, and so have I in obtaining the results below.
- With ln(maximum force) the dependent variable and ln($\frac{1}{2}.m.v^2$) the independent variable, the slope was estimated as 0.26. If the relationships in section 4.5 of this book are relevant, maximum force will be proportional to $m^{n/(n+1)}.v^{2n/(n+1)}$ (see section 17.11.1). Thus 0.26 is interpreted as an estimate of n/(n + 1), and the exponent n is estimated as 0.35.
- Maximum displacement S in section 4.5.1 is proportional to $(m.v^2)^{1/(n+1)}$. It may be asked whether this is true for the measures of deformation reported by Zhu, Liu, et al.
- If n is 0.35, 1/(n + 1) is 0.74. Of the measures of deformation reported by Zhu, Liu, et al., the one that is most closely linked to maximum force is probably $W_{uf}$, the permanent upper surface deformation at the impact position. For this, the regression estimate of the exponent was 0.72, which is close to 0.74.
- In the case of four other measures of deformation, the regression estimates of the exponent were in the range 0.51 to 0.87.

- Zhu, Liu, et al. split $W_{uf}$ into global deformation and local deformation. (For pipe of type B, global deformation tended to be the greater.) Considering these separately, I find regression estimates of the exponents to be 0.85 and 0.51.

It was said in section 4.2.4 that Jeong et al. (2011) regarded n = 0.5 as an appropriate assumption, based in part on the theoretical work of Wierzbicki and Suh (1988). Wierzbicki and Suh were concerned with the deformation of tubes. I am unsure whether that work is relevant to that of Zhu, Liu, et al., but an estimated exponent of 0.35 is not very different from 0.5.

Is impact energy ($\frac{1}{2}.m.v^2$) indeed the important factor in determining outcomes?

- In regressions with both ln(m) and ln(v) as independent variables, the coefficient of ln(v) was roughly twice the coefficient of ln(m): the ratio was close enough to 2 to give some support to the proposition, but far enough from 2 for this to be not fully convincing.
- In regressions with ln($\frac{1}{2}.m.v^2$) and ln(v) as independent variables, the confidence interval of the coefficient of ln(v) did not include 0 in three of the six cases (and in the other three cases the data had sufficient scatter that the confidence interval was very wide).

Thus it is a reasonable approximation to describe $\frac{1}{2}.m.v^2$ as the important factor in determining outcomes, but there is some evidence that this does not fully capture the effects of m and v.

Figure 6(b) of Ryu et al. (2017) reports a series of drop tests on pipes using a flat impactor, and gives the relationship between impact speed and the resulting strain. When v increased by a factor of 3.2, strain increased by a factor of 2.1. Thus strain was approximately proportional to $v^{0.65}$.

Readers interested in undersea pipelines may also be interested in impact of a ship's bow with a rigid wall: see section 4.2.4.

## *17.13 Impact with a beam*

Alves and Yu (2005) report results obtained from a finite element model of a mass impacting a fully clamped beam (aluminium or mild steel). They employed several material models. The results here refer to the model described as real (a curved stress-strain relationship). Maximum deflection and the associated time (similar to pulse duration, though not the same) will be considered here.

Results for aluminium included the following.

- When impact mass increased by a factor of 5, maximum deflection increased by a factor of 1.75, and the associated time increased by a factor of 1.66.

- If the proportionality relationships for S and T in Table 4.2 are assumed to apply here, these factors would respectively suggest that exponent n is about 1.9 or about 2.2. Thus the two results are consistent.

Results for mild steel included the following.

- When impact mass increased by a factor of 2.5, maximum deflection increased by a factor of 1.63, and the associated time increased by a factor of 1.57.
- If the proportionality relationships for S and T in Table 4.2 are assumed to apply here, these factors would respectively suggest that exponent n is about 0.9 or about 1.0.
- When impact velocity increased by a factor of 1.4, maximum deflection increased by a factor of 1.39, and the associated time changed very little.
- If the proportionality relationships for S and T in Table 4.2 are assumed to apply here, both these findings would suggest that exponent n is about 1.0.
- Thus the four results are consistent.

Sonoda et al. (2012) report on the response of a reinforced mortar beam impacted by dropping 100 kg from 0.2 m or 0.6 m. Comparing Figures 7 and 9 of that paper, it seems that multiplying drop height by 3 leads to multiplying maximum displacement by approximately 3. If the proportionality relationships for S in Table 4.2 are assumed to apply here, exponent n is about 0.

Srivastava and Rehkugler (1976) report the results of impacts of a pendulum with a cantilever beam. They give various plots on logarithmic axes. The straight lines imply maximum deflection is a power function of mass and of velocity of impact, the exponents being respectively about 0.8 (their Figure 6) and about 2 (their Figure 7). If the proportionality relationships for S in Table 4.2 are assumed to apply here, exponent n is a little greater than 0.

## *17.14 Axial impacts with tubes*

Tubes are used as structures, and specifically as impact energy absorbing structures. In impact, they fold progressively, and force is reasonably constant. Velmurugan and Muralikannan (2009) conducted impact experiments at speeds between 7.0 m/sec and 8.3 m/sec, and reported crush distance and peak load for circular, square, and rectangular tubes. They had in mind vehicle crashes. (For crushing of cars, see section 17.9.)

I have examined how ln(crush distance) and ln(peak load) depend on ln(v). I have calculated values of n based on the assumption that the proportionality relationships will be the same as for S and $m.A_{max}$ in section 4.5. Results for the two dependent variables are quite different.

- For circular, square, and rectangular tubes, exponent n calculated from the results for crush distance is between 0.4 and 0.
- Exponent n calculated from the results for peak load is between 1.3 and 6.

The difference seems to arise because crush distance reflects the whole impact; in contrast, in these impacts peak force occurs very early (Figures 8, 11, 14 of Velmurugan and Muralikannan), not at almost the same time as maximum crush. Velmurugan and Muralikannan discuss the crushing, including the folding of the tubes, in some detail.

## *17.15 Fibre-reinforced composites*

Isa et al. (2014) report on drop tests conducted on fibre-reinforced composites. Their Figure 3 gives the peak load at three impact energies for several materials. The data appears to show that different materials vary quite considerably in the peak load for a given impact energy, and that they also vary in how much the peak load increases by when impact energy increases.

- In the case of glass fibre reinforced polyester (for example), I find that peak load was approximately proportional to $v^{0.7}$, corresponding to exponent n being about 0.5.
- In the case of Kevlar composite, I find that peak load was approximately proportional to $v^{0.3}$, corresponding to exponent n being about 0.2.

A difference in n is a way of interpreting the apparent variation between materials in sensitivity of peak load to impact speed. However, the confidence intervals associated with the exponents are very wide. (That is not surprising, as there were only three impact speeds.)

Zouggar et al. (2016) conducted drop tests on a laminate composite material made of polyester matrix reinforced with S-glass fibres. Doubling of drop height led to maximum force being multiplied by 1.39 and maximum bending being multiplied by 1.47. These factors respectively imply that exponent n is about 0.9 or about 0.8.

In Table 3 of Martínez et al. (2012), maximum force is reported from drop tests with v and m both varying. Regressing ln(maximum force) on ln(v) and ln(m), the slopes are found to be approximately 1.0 and 0.5. Thus exponent n is about 1.

In Figure 4 of Spronk et al. (2018), the increase of maximum force with maximum displacement, for four materials separately, suggests exponents of between 0.5 and 1.0.

Jang et al. (2002) report on damage to carbon fibre reinforced plastic laminates impacted by soft and hard projectiles.

- For hard projectiles, they consider that kinetic energy will be proportional to a weighted average of $F_{max}^2$ and $F_{max}^{5/3}$ (see section 4.2.4).
- The soft projectiles that they have in mind are birds striking aircraft. These, they consider, will behave as a liquid, and maximum force will be proportional to the square of impact velocity.

Jang et al. plot a measure of damage versus energy and versus maximum force. The force corresponding to zero damage is about the same for soft and hard projectiles, and it therefore seems appropriate to describe it as a critical value for the initiation of delamination. The energy corresponding to zero damage, on the other hand, was much less for hard projectiles than for soft projectiles.

For penetrating damage of a woven glass/epoxy composite, see section 16.6.

## *17.16 Metal honeycomb*

Xie et al. (2018) carried out drop tests on to titanium honeycomb sandwiched between titanium facesheets. Impact speeds were between 1.4 and 2.8 m/sec.

I have regressed ln(maximum force) on ln(v), and found the slope to be about 0.7; the slope for dependence of ln(S) on ln(v) is about 1.5. The corresponding values of exponent n are about 0.5 and about 0.3. (In both cases, the confidence interval is wide.)

## *17.17 Liquid in a bag*

A liquid may be packaged in a bag. Impact tests of a liquid in a bag are rather unusual, as far as I know.

Watanabe et al. (2009) reported drop tests of bags containing a liquid.

- Three measures of liquid speed were reported: average swirl speed, maximum swirl speed, and average rising flow speed. It is not clear what relationship best describes how these measures depend on drop height. Watanabe et al. appear to suggest different forms of the relationship for the three measures: approximately constant, proportional to the square root of drop height, and proportional to drop height.
- Maximum pressure at two locations in the bag was reported. Dependence of maximum pressure on drop height was quite weak; it was rather stronger in the tests of Shinoda et al. (2008).

Both papers mentioned show interest in using water hammer theory to explain some aspects of the results.

## *17.18 Shear thickening fluid*

Fu et al. (2018) reported experiments in which a shear thickening fluid was impacted.

Figure 5(b) of Fu et al. shows instantaneous force vs. instantaneous penetration depth for various impact speeds. Two features of this are that, before maximum force is reached, force is approximately the same function of penetration depth, whatever the impact speed was; and force is approximately proportional to $x^{0.7}$, where x is penetration depth at any moment.

Results for maximum force and penetration were as follows.

- Maximum force, plotted in their Figure 6(a), is approximately proportional to $v^{0.8}$, where v is impact speed.
- Maximum penetration depth, plotted in their Figure 6(b), is approximately proportional to $v^{1.1}$.

According to the proportionality relationships in section 4.5.1, the exponents would respectively be 0.82 and 1.18 if n were 0.7. Thus the three exponents are consistent.

# 18. Damage to fruits, vegetables, and eggs

## *18.1 Introduction*

Impact may damage farm produce during any of many operations before it reaches the consumer. At the level of the broad topic, sections 18.2 and 18.3 will respectively note some similarities with and some differences from research on blunt injury of humans, section 18.4 will comment on the use of direct measures of damage, and section 18.5 will discuss the biofidelity (or otherwise) of the instrumented spheres used to collect impact data. Then at the level of a research project, sections 18.6 - 18.13 will sketch some of the types of experiment conducted and results found.

Among the most important research methods is use of a pseudo-fruit or instrumented sphere to measure accelerations. There will be some examples later in this section. Some features of examples elsewhere in this chapter are as follows.

- Bruising, and other direct measures of damage: sections 18.4, 18.12.
- Biofidelity: section 18.5.
- Dropping of boxes with accelerometer: section 18.6.
- Instrumented sphere, small fruits: sections 18.7, 18.8.
- Impact a fruit with a sensor of low mass: section 18.9.
- Finite element model: section 18.10.
- Drop an egg with an accelerometer on it: section 18.11.
- Electronic egg: section 18.13.

The examples of produce mentioned are as follows.

- Peaches: section 18.1.
- Bell peppers (capsicums): section 18.1.
- Satsuma mandarins: section 18.1.
- Nijisseiki: section 18.1.
- Pumpkins: section 18.1
- Giant radishes: section 18.1.
- Apples: sections 18.1, 18.13.
- Seeds and grains: section 18.4.
- Strawberries: section 18.6.
- Mangos: section 18.6.
- Blueberries: sections 18.7, 18.8.
- Pears: section 18.9.
- Potatoes: section 18.10.
- Eggs: sections 18.11, 18.12, 18.13.

Genge et al. (1978) report tests of several materials that might be used as cushions of fruit-catching surfaces. They had in mind the harvesting of peaches by a shake-catch method. They did not use peaches or any other fruit in their experiments, but instead a hard rubber ball (described as similar to the average size peach) was used as a pseudo-fruit. An accelerometer was attached to it. For a drop height of 2.44 m, as an example, the maximum acceleration for eight types of foam cushion ranged between 62 g and 310 g. Genge et al. attribute some of the high accelerations to bottoming out. An air mattress gave rather better results, maximum accelerations of between 34 g and 52 g for different air pressures. For most foams, coefficient of restitution decreased with increasing drop height. In the Hunt and Crossley model (section 4.3), coefficient of restitution does not depend on impact speed (and thus drop height).

Marshall and Brook (1999) used an instrumented sphere (that determined velocity change and maximum acceleration) to identify where the greatest potential for damage of bell peppers (capsicums) occurs. They attempted to mimic how pickers harvest peppers into plastic pails, how they empty the pails into wooden pallet bins, and how the peppers move through the grading line at a packing shed. The observation of chief interest was acceleration as measured by the instrumented sphere, and the circumstances in which high values of this were observed. A threshold of acceleration measured in this way that corresponds to pepper damage was available from earlier literature. Velocity change was also measured by the instrumented sphere. For the grading line (as an example), Marshall and Brook presented their results as plots of maximum acceleration versus velocity change, using different symbols for different places along the line. Only a few data points lie above the presumed threshold for bruising. It can be seen on the scatterplot what the locations on the grading line were, and that the velocity change (of the instrumented sphere) was above 1 m/sec. It may be added that maximum acceleration is approximately proportional to velocity change.

Data analysis has sometimes employed power functions to describe impacts of fruits.

- For satsuma mandarins falling on to an iron plate, Iwamoto et al. (1976) reported linear dependence of $\ln(A_{max})$ on ln(drop height), the slope being 0.89. According to the relationship in section 4.5.1, that implies exponent n is about 8.
- For the impact surface being a concrete floor, Chuma et al. (1978) reported a similar relationship, the slope being 0.72, which implies exponent n is about 3.
- A possible reason for the use of power functions was familiarity with the Hertzian impact theory mentioned in sections 4.2.4 and 4.3.2 (Chuma et al., 1970, 1978). For this, the slope will be 0.6.
- Kawano et al. (1984) knew of that prediction of Hertzian impact theory, and they compared it with an exponent of 0.43 that they found for Nijisseiki. (The papers by Kawano et al. and Chuma et al., 1970, are in the Japanese language, which I do not understand. I

apologise to readers and to the authors if I have misunderstood anything.)

- In addition, Chuma et al. (1970) report data for apples falling on to either plastic tile or corrugated board. Plotting ln($A_{max}$) vs. ln(drop height), I find the relationships are good straight lines, with slopes of 0.6 and 0.8. These respectively imply that exponent n is 1.5 (as for Hertzian theory) and 4.
- Rider et al. (1973) also were familiar with theory. Figure 2 of Rider et al. (1973) refers to impact of a pseudo-fruit with three surfaces. For each surface, what is shown horizontally on a logarithmic scale is proportional to $h^{0.2}$ (where h is drop height). Plotting $A_{max}$ vertically on a logarithmic scale, a straight line of slope 3 is therefore expected. That was found empirically for one of the surfaces; the covering of the pseudo-fruit apparently bottomed out on the other surfaces.
- In parts (a) and (b) of their Figure 3, Komarnicki et al. (2017) show power functions for the dependence of force on drop height of apples. The exponents are between 0.45 and 0.65. Komarnicki et al. make brief mention of Hertzian theory.
- Chen et al. (1993) made the point that Hertzian theory could be applied as follows. Suppose that it is desired to measure the firmness of produce (pumpkins, and giant radishes, for example) with a hammer impact method. As maximum force is proportional to $v^{1.2}$, the ratio of maximum force to $v^{1.2}$ is a measure of firmness. This is useful if impact velocity can be measured more accurately than it can be controlled.

## *18.2 Similarities with human injury*

There are several ways in which research on damage to agricultural produce resembles research on human injury.

- It is common to consider that maximum acceleration is useful as a proxy for damage, to measure it experimentally, and to plot it as a function of drop height or impact speed. Velocity change is sometimes also measured.
- There is a desire to know in what circumstances damage is most likely to occur. This means, for example, where in a system of conveyor belts, or where in the whole logistics chain.
- To measure maximum acceleration, and to identify where damage is likely to occur, instrumented spheres have been invented, and are inserted into the chain of processing.
- There is awareness of the undesirability of a cushion bottoming out. And there is concern that some cushions are not stiff enough, and do bottom out.
- There is probably quite broad awareness of some aspects of theory. There is some interest in more advanced theory. For example, for modelling impact, Peleg (1985, pp. 850-852) proposed two nonlinear

springs (one hardening according to a cubic equation, and one softening according to a cubic equation), a viscous damper, and a Coulomb damper. And Li et al. (2017) review a number of impact models in the context of impacts of fruits. Their Table 3 lists eight models of impacts between two soft spheres, and their Table 4 lists eight models of the drop of a soft sphere on to a rigid plate. Both lists include the Hunt and Crossley equation.
- However, as far as I can see, there has not been properly productive integration of theory and experimentation. In my opinion, much experimentation is not as useful as it might be. Very often, it is not very systematic in either the input variables or the outputs. Very often, there simply isn't much data collected.
- In my opinion, much reporting of experimentation is not as useful as it might be. Information is missing, details are not clear, there seem to be misprints, some data is omitted, possible anomalous observations are not discussed, and so on.

## *18.3 Differences from human injury*

There are several ways in which research on damage to agricultural produce differs from research on human injury.
- There is more concern with the very common low levels of damage, because any damage reduces the commercial value of the produce substantially.
- Surprisingly, there seems to be little concern with physical properties of instrumented spheres: the mass, and the surface stiffness, in particular. See section 18.5 below for some discussion of this.
- HIC (the Head Injury Criterion) is not relevant, of course. Thus interest in algebraic properties of HIC, and comparisons of HIC with maximum acceleration, are absent. Indeed, it is not common to use physical measurements other than maximum acceleration, or perhaps maximum force, as proxies for damage.
- In contrast, direct measures of damage are used, such as bruise area or bruise volume on a fruit or vegetable. (Separation of a physical proxy for injury from the injury itself is necessary in the context of human injury because tests in which a human is injured cannot be conducted.) There is discussion of this in section 18.4.

## *18.4 Direct measures of damage, as contrasted with proxies*

The practicability of directly measuring damage means that the results are of immediate appeal. However, although my limited knowledge makes me reluctant to comment too strongly, the following line of argument makes me think that other approaches are needed also.

- It seems to me that bruising (or cracking, or other damage) is an extra layer of difficulty. Not only is a theory needed about physical measures such as maximum acceleration, maximum force, or maximum displacement, but a theory is also needed about the bruising, and perhaps even about classifying the severity of bruising. This includes the time after the impact at which bruising is assessed: no damage after 1 hour does not necessarily imply no damage after 1 week.
- Also, measures of damage refer to the fruit or vegetable. That may be of secondary interest --- chief interest is often in the surface or cushion that is impacted. It is not clear that knowledge of fruit bruising is as useful as maximum acceleration (for example) in understanding the cushion.

An example of research in which amount of bruising was classified is Lu and Wang (2007). A reason for mentioning this paper is that it employed the concept of damage boundary curves described in section 17.5. Depth of damage of apples subjected to drop tests will be briefly discussed at the end of section 18.13.

Seeds and grains may be damaged by impacts, just as peaches and peppers may be. I do not know for sure, but I doubt if there are instrumented seeds and grains in the same way that there are instrumented spheres. My impression is that research relies on direct measures of damage such as proportion of seeds seen on inspection to be damaged (e.g., ruptured or cracked), and that physical proxies such as acceleration or force are not used. An example of research that used both visible damage and germination is Berry et al. (2014). A reason for mentioning this paper is that, unusually, damage was desirable: the topic of study was deliberate damage of weed seeds when harvesting wheat.

There is mention at several points in this book that it is difficult to know what "really" causes human injury (whether it is acceleration or force or deformation or something else). See, for example, sections 3.3 and 5.2. My impression is that experts are equally vague about the causes of damage to fruit and vegetables.

## *18.5 Biofidelity of the instrumented sphere*

For a system of handling or transporting agricultural produce, accelerations are measured by an instrumented sphere being included in the field-to-plate line.

Herold et al. (1996) listed technical requirements of an instrumented sphere. The first of these was "to correspond with the real produce's size, shape, mass, and elasticity". Cerruto et al. (2015) noted that the greatest drawback of instrumented spheres "is mainly due to the considerable differences between real and artificial fruit, largely restricting the transferability of measured impact data (force or acceleration) to real products". The accelerations are only correct if the instrumented sphere resembles a real fruit (or vegetable) in respect of important characteristics such as mass and surface stiffness. Praeger et al. (2013) made the point as follows: "For determination of realistic mechanical loads inflicted on the produce, measuring devices which are similar to the real products concerning the geometrical shape and the physical properties like the elastic compliance are necessary."

It might be said that for some purposes, accelerations that are correct in an absolute sense may not be needed, e.g., when the aim is to identify the points on a processing line where acceleration is highest. That is a valid point, but even so, I am surprised that more attention is not given to what needs to be realistic in what circumstances. I was able in the previous paragraph to give some references in support of the desirability of biofidelity, but most authors seem unconcerned.

It is possible that there is more than one reason why bruising of a fruit or vegetable occurs (for example, acceleration or force or deformation). A hypothesis about which is most important would carry implications for what characteristics of the instrumented sphere need to be realistic. In a sense, all or most of the reasons might be merged into "too fast relative to the stiffness of the surface struck". But it is likely to be worth distinguishing between speed and stiffness as reasons, and angle of impact might be a third.

Measuring something different from what is most important is not necessarily a mistake. Specifically, the use of surface bruising as a direct measure of damage seems to imply that acceleration (measured at approximately the centre of gravity) is not thought of as really causing fruit damage. It seems likely to me that surface deformation of the fruit is thought of as the real cause of damage, with acceleration being more convenient to measure and highly correlated with deformation. Section 5.2.2 considered maximum deformation of the human head, and found it to be a power function of maximum acceleration and otherwise not dependent on stiffness of the car bonnet and impact speed. In the present context, therefore, maximum deformation of a fruit is a power function of maximum acceleration, and otherwise not dependent on stiffness of the surface struck and impact speed.

## *18.6 Kitazawa et al. (2014)*

Kitazawa et al. (2014) report tests of two materials that might be used for cushioning strawberries during transport. They dropped boxes, instrumented with an accelerometer and having one or other of the materials on the floor of the box, from heights of between 5 and 30 cm. They reported velocity changes and maximum accelerations.

I have plotted ln(maximum acceleration) versus ln(velocity change). The relationship is approximately a straight line for both materials, the slopes being 1.4 and 1.3. The exponents n are thus 2.4 and 1.7.

Nakanishi et al. (2015) report a similar experiment with a fibreboard box containing two mangos and an accelerometer, drop height h being between 2.5 cm and 40 cm. Nakanishi et al. fit a power function, $A_{max} \propto h^{0.62}$. (This suggests that ln(maximum acceleration) versus ln(velocity change) would have a slope of 1.2.)

For dropping a box, see also section 17.7

## *18.7 Xu et al. (2015), and Yu et al. (2014)*

It is difficult to adapt instrumented sphere methods to small fruits. Nevertheless, results obtained with an artificial blueberry were reported by Xu et al. (2015, Table 2).

- Doubling drop height from 15 cm to 30 cm multiplied $A_{max}$ by 1.62.
- Doubling drop height from 30 cm to 60 cm multiplied $A_{max}$ by 1.50.
- The same ratio of drop heights leads to (approximately) the same ratio of maximum accelerations, therefore the relationship is approximately a power function.
- The exponent is 0.64, implying that n is about 1.8.

The drops were on to a hard plastic surface. Presumably, then, the artificial blueberry deformed, not the surface, and the estimated n represents a property of the artificial blueberry, not of the hard surface, and not of a real blueberry. Xu et al. use bruising rate as the outcome measure for real blueberries; bruising rate in drop tests, together with $A_{max}$ in drop tests, is used to interpret $A_{max}$ in packing line observations.

For an artificial blueberry falling on to two surfaces, Yu et al. (2014) plotted maximum acceleration versus drop height (which varied by a factor of 8 from smallest to largest). They interpreted the relationships (their Figure 5(a)) as straight lines (but not lines of proportionality). If the relationships are interpreted as power functions, the exponent is about 0.4 when impact was with the hard plastic surface (and exponent n is about 0.8), and about 0.8 when impact was with the "No Bruze" urethane foam surface (and exponent n is about 4).

## 18.8 Xu and Li (2015)

Xu and Li (2015) paid more attention to properties of the artificial fruit than many authors do. They made some comparisons of the artificial blueberry used by Xu et al. (2015), a second-generation artificial blueberry, and real blueberries, in respect of both mass and surface firmness. (The second artificial blueberry has smaller mass and lesser stiffness than the first.) Figure 14 of Xu and Li shows $A_{max}$ versus drop height, for the four combinations of the two artificial blueberries and two surfaces on to which they fell.

The following results are for a steel surface.

- Effect of drop height, first artificial blueberry. Xu and Li show a straight line relationship with a non-zero intercept. A plot of $\ln(A_{max})$ versus $\ln(h)$ is also a good straight line. The slope is 0.90. (Other examples of two alternative relationships both being approximate straight lines occur in sections 16.2 and 18.12.)
- Effect of drop height, second artificial blueberry. Xu and Li show a straight line relationship with a non-zero intercept. A plot of $\ln(A_{max})$ versus $\ln(h)$ is also a good straight line. The slope is 0.68.
- $A_{max}$ is lower for the second artificial blueberry than for the first. Xu and Li gives the masses of the two types of artificial blueberry, and also the compression force required for a 1 mm deformation. It might be that this can be interpreted as proportional to stiffness k. It might also be that steel is sufficiently stiff to have negligible effect on the stiffness of the system. If these assumptions are correct, m/k is greater for the second artificial blueberry than for the first by a factor 2.36. And on that basis, $\ln(A_{max})$ may be regressed on both $\ln(h)$ and $\ln(m/k)$. The estimated coefficients were 0.79 and -0.47. If exponent n were 2, for example, these would be 0.67 and -0.33, not very different.

The following results are for a padded surface.

- Accelerations were appreciably lower than for the steel surface, and it would be difficult to read them from the scatterplot.
- There was very little difference between results for the two artificial blueberries. This suggests that the stiffness of the padding was intermediate between the stiffnesses of the two artificial blueberries. (If padding stiffness were high, the results would be different for the two artificial blueberries, as the results were for a steel surface. If padding stiffness were low, the results would be different for the two artificial blueberries, as they were of different masses and their stiffnesses would be irrelevant in the case of low padding stiffness.)

If indeed it is correct that the stiffness of the system sometimes reflects the stiffness of the artificial blueberry (e.g., when the first artificial blueberry hits steel) and sometimes reflects the stiffness of what it hits (e.g., when the first artificial blueberry hits the padded surface), that must make the interpretation of the data difficult.

Figure 6 of Takeda et al. (2017) shows that the nature of the impact surface makes an enormous difference to maximum acceleration corresponding to given drop height (or to the drop height that corresponds to given maximum acceleration). The surfaces considered ranged from steel to a suspended fabric net.

## *18.9 Chen et al. (1996)*

### 18.9.1 Introduction

The paper by Chen et al. (1996) is about impacting a fruit with a sensor of low mass, in order to measure the fruit's firmness. I found it a little odd that the paper presents theoretical results that are based on an assumption of Hertzian impact (see sections 4.2.4 and 4.3.2 of this book) and thus involve power-function relationships, but then analyses data with straight-line regressions.

I will now use power functions to summarise some of the results in Table 1 of Chen et al. These refer to Bartlett pears, with an impactor of mass 10 gm.

- The dependent variable is the "Firmness Index" (FI), which is the ratio of $A_{max}$ to the duration T of the pulse. Equations in Table 1 of Chen et al. permit calculation of FI at the four combinations of two values of each of two explanatory variables.
- The first explanatory variable is drop height (h), which was 2 cm or 4 cm.
- The second is "firmness", the elastic modulus (Emod), which was measured for each fruit. I calculated FI at Emod = 1.5 MPa and Emod = 4.5 MPa. (In these experiments, Emod has a range of values from slightly below 1 MPa to slightly above 5 MPa.)

From the results in section 4.5 and Table 4.2 of this book, FI (this is $A_{max}/T$) is predicted to be proportional to $(m/k)^{-2/(n+1)}.v^{(3n-1)/(n+1)}$.

- Thus in terms of h, FI is predicted to be proportional to $h^{(3n-1)/(2n+2)}$.
- I will treat Emod as being similar to k. (Chen et al. considered Poisson's ratio to be a constant for their fruits.) Thus FI is predicted to be proportional to $Emod^{2/(n+1)}$.

### 18.9.2 Results

The regression equations give the following results.

For h = 2 and Emod = 1.5: FI is 8.1.

For h = 4 and Emod = 1.5: FI is 12.5.
For h = 2 and Emod = 4.5: FI is 13.8.
For h = 4 and Emod = 4.5: FI is 20.3.
These values of FI are the starting point for estimation of exponents.

Exponents may be calculated as follows.
Exponent of h, at Emod = 1.5, is 0.63. (This is calculated as ln(12.5/8.1) / ln(4/2).)
Exponent of h, at Emod = 4.5, is 0.56.
Exponent of Emod, at h = 2, is 0.48.
Exponent of Emod, at h = 4, is 0.44.

The four estimates of n are therefore as follows.
From change of h at Emod = 1.5: n is 1.3. (If (3.n - 1)/(2.n + 2) is 0.63, n is 1.3.)
From change of h at Emod = 4.5, n is 1.1.
From change of Emod at h = 2, n is 3.1. (If 2/(n + 1) is 0.48, n is 3.1.)
From change of Emod at h = 4, n is 3.6.

The estimates of n based on drop height (1.3 and 1.1) are not very far from 1.5, the value for Hertzian impact.

The estimates of n based on "firmness" (elastic modulus Emod) are considerably different. Possibly the experimental method of measuring this was not appropriate, or perhaps I should not interpret it as similar to k.

## *18.10 Cerruto et al. (2015)*

Cerruto et al. (2015) report a simulation study using a finite element model. They were able to vary drop height, diameter, density, and Young's modulus (stiffness). Mass naturally varied with diameter and density. Properties were chosen to be similar to those of potatoes. They observed maximum impact force and maximum acceleration at the centre of the simulated potato.

There are thus a lot of results in the paper. I did not, however, notice any comment about whether the results were at all similar to what would be expected from Hertz contact theory (see section 4.2.4).

## *18.11 Zhang et al. (2015)*

In experiments by Zhang et al. (2015), eggs were dropped from low heights (insufficient to break them) on to three cushioning materials, EPE (expanded polyethylene), EPS (expanded polystyrene), and wall board.

Maximum force and impact time were measured with an acceleration sensor mounted on the egg.

- As drop height changes, maximum force will change in the same way as maximum acceleration (see section 4.5).
- I have calculated exponent n from the proportionate increase in maximum force relative to the doubling of drop height from 10 cm to 20 cm.
- The values of n were 2.9 for EPE, 1.2 for EPS, and 1.6 for wall board.
- Even though duration of impact was reported to an accuracy or precision of 0.1 msec, I doubt if it is appropriate to calculate the corresponding n. Nevertheless, it may be noted that for each cushioning material, duration decreased with increasing drop height. This suggests n exceeds 1 in each case (see section 4.5), which is in agreement with the findings from maximum force.

## *18.12 Anderson et al. (1970)*

In earlier sections (5.4.3, 6.7.4) I mentioned the possibility that a model of impact might include a mass being put into motion. For example, some fraction of the mass of a bonnet might move when a pedestrian headform strikes it, or some fraction of the mass of the body wall might move when struck by a robot or a cricket ball or a less-lethal projectile.

Anderson et al. (1970) used such a model in the context of the cracking of hens' eggs. (Carter, 1970, summarised the results.) They described experiments showing that the cage floor mass is an important factor. The mass was varied by attaching a small magnet. Suppose that $M.v^2$ is the quantity determining whether cracking occurs (M = the unknown mass of the cage floor, v = impact speed). This is proportional to M.h (h = drop height). Thus for a constant incidence of cracking (e.g., 50 per cent), the reciprocal of the corresponding drop height 1/h is proportional to M. And M is the sum of the mass of the floor alone (M*, say) and the mass x of the magnet that has been added to it. Thus 1/h has linear dependence on the experimental variable x, with an intercept interpretable in term of M*.

According to Carter (p. 552), "the feature of the floor that determines its resistance to acceleration is its effective mass". The model thus seems to be one of putting a mass into motion. However, it is rather different from what was described in sections 5.4.3 and 6.7.4. In those sections, the principle of conservation of momentum is used, and putting the mass into motion occurs at a stage before damage does.

Anderson et al. discuss the idea that what matters is the momentum of the floor relative to the egg, rather than the kinetic energy, and reject it. They do so because the plot of 1/h versus x is a very good straight line. I do not regard that as conclusive: a plot of $1/\sqrt{(h)}$ versus x is also a very good straight line (the correlation coefficient is bigger than 0.99). This suggests that what matters is momentum. (Other examples of two alternative relationships both being approximate straight lines occur in sections 16.2 and 18.8.)

## *18.13 van Mourik et al. (2017)*

van Mourik et al. (2017) reported experiments in which an "electronic egg" on a pendulum struck either an egg that was (in effect) clamped in position or a metal anvil. Table A1 of van Mourik et al. gives $A_{max}$ measured by the electronic egg for various energies of impact.

I have calculated impact speed v from impact energy, and I have plotted $\ln(A_{max})$ versus $\ln(v)$.

In section 5.2.2, it was noted that if two power-function springs have the same exponent n and are in series, they are equivalent to a single power-function spring having the same exponent n. For example, if n = 1, the reciprocals of the k's are additive.

Features of the data are as follows.

- In both cases, the slope was approximately 1. That implies exponent n is approximately 1.
- Assume that n = 1, and that the metal anvil is much stiffer than the electronic egg.
- Then the stiffness of the electronic egg can be calculated, except for a constant of proportionality, from the ratio $A_{max}/v$ observed in impacts with the metal anvil.
- The stiffness of the electronic egg and the egg in series can be calculated, except for the same constant of proportionality, from the ratio $A_{max}/v$ observed in impacts with the egg.
- Consequently, the ratio of the stiffness of the electronic egg to the average stiffness of the eggs used can be calculated. I find that it is about 0.57.

It seems likely that similar reasoning could be applied to data from drop tests of apples that was reported by Unuigbe and Onuoha (2013). No electronic apple was used, and thus accelerations were not measured, but depth of damage was measured. For the metal impact surface, presumably all the deformation was of the apple. Other impact surfaces were less stiff. As the plots of ln(depth of damage) vs. ln(drop height) might be interpreted as straight lines of slope 0.5, exponent n is about 1, and for springs in series the reciprocals of the k's will be additive. Consequently, for the various impact surfaces, it should be possible to estimate the ratio of surface stiffness to apple stiffness.

# Part D: Ending and Appendices

Part D has the following chapters.

19. Average performance in a variety of conditions.
20. Concluding comments.

The following are the Appendices.

21. Appendix 1: Bottoming out.
22. Appendix 2: Improvements in the design of cars for pedestrian impacts.
23. Appendix 3: Is use of an acceleration-based proxy for injury evidence that acceleration is the most important factor?
24. Appendix 4: Effect of random variation on the estimate of n.
25. Appendix 5: Comparison of three estimates of n.
26. Appendix 6: Stiffness when there are several speeds of impact.

# 19. Average performance in a variety of conditions

## *19.1 Introduction*

### 19.1.1 Some questions and answers

Testing is an important method of improving safety. But a test is typically conducted in closely-specified conditions, whereas there is a great variety of conditions in the real world. It is to be hoped that the result of the test is also relevant to the real world. In principle, this might apply to many other types of test as well as to impact testing. The purpose of this chapter is to show how generalisation from a test result might be made. Some questions and answers might be helpful at this point.

- What is the starting point? A result obtained in closely-specified test conditions.
- What is the aim? To calculate the likely performance in the real world. There are two aspects to this: performance in conditions other than those in the test, and the averaging of performance over all conditions that occur.
- Does the concept of an average have any particular implications? Yes, it means (a) that there must be a numerical quantity that can be averaged, and (b) that it is appropriate to base decisions on this quantity. Such a quantity is often referred to as a utility (or a cost, or a value).
- How will the average utility be calculated? It will be based on the utility in a given set of conditions together with the probability of that set of conditions occurring in the real world, and consideration of all sets of conditions.
- Are the relative frequencies of different sets of conditions known? In principle, yes, there is some information available about the conditions (e.g., speeds) of real-world accidents and their frequencies.
- Is it likely that a utility will be recorded in the test? No, it is much more likely that something that is convenient for the physical process of measurement will be chosen. The measurement will need to be converted to the corresponding utility.
- Is the physical measurement known for every set of conditions? No, it is known only for the set of conditions used in the test. Many tests might be performed in order to cover the range of conditions that occur, or some theory might be available to generalise from one set of conditions to others.

The idea in this chapter was largely my colleague Robert Anderson's, and it occurred in the context of pedestrian headform impact testing (see chapter 2). I put it into the form used in section 19.3.2. Our first account of the method was as Hutchinson et al. (2012), and this chapter is based on a later conference paper (Hutchinson et al., 2016). Chapter 20 of Hutchinson (2018) is similar to this chapter, and there is more on the topic in chapters 21 and 22 of Hutchinson (2018).

This chapter will sometimes be written in terms of impact testing specifically, and sometimes in more general terms. For example, HIC and speed are specifically referred to. More generally, they would be the output obtained in the test, and the input that is specified for the test but varies in the real world.

### 19.1.2 Approaches to testing

At present, the testing system seems to presume that the result at one choice of speed and other conditions is sufficient. This appears simple, but that appearance may be illusory. There are some difficulties that are hidden rather than non-existent, in particular concerning the choice of the single set of conditions for testing. Some people may say that a low test speed should be chosen because low speed impacts are far more common than high speed impacts, others may say that a high test speed should be chosen because it is the fatal and near-fatal impacts that are of most concern, and still others may argue for a typical impact speed in the middle of the distribution of real accident speeds. (Of course, many other factors are taken inro account, notably ones of practicability.)

Alternatively, we might wish to know the level of safety in a wide range of real-world impact scenarios, and to have some sort of average available. The aim in this chapter is to propose a method of doing this.

### 19.1.3 Outline of method

The method is reasonably straightforward in principle. The following issues will be important.

- The need to do lots more tests in order to obtain the basic data on how HIC varies with speed --- or, alternatively, a theory about this is required.
- While it is hoped that HIC reflects or indicates likely injury, it is not itself a measure of injury.
- It is unlikely that it is sensible to average HIC. One person with HIC = 900 and one with HIC = 1100 is about as bad an outcome as two people with HIC = 1000, but this simple averaging is arguably inappropriate for one person with HIC = 200 and one person with HIC = 1800. (That is, HIC is not truly quantitative, but more like an ordinal variable.)
- Despite the previous point, some sort of average or summary measure will be needed. That means that information about

frequencies of impacts at different speeds will be required as input to the calculation.

One possible method (and perhaps it is the best method) of determining the level of safety in a range of scenarios is to test across the range of scenarios and combinations of scenarios. In some cases, something like this is indeed done: many different locations on a car exterior are tested. Nothing said below should be taken as critical of that straightforward approach. But it may be possible to use theory to economise on the number of tests.

The following sentences refer to motorcycle helmet design, and appear in Gilchrist and Mills (1994, p. 217). "The compromise design should attempt to minimise the total harm to helmet wearers. The injuries predicted for a specific impact velocity and impact object should be weighted according to the frequency distribution found in accident surveys." That seems very similar to the aim expressed above. What will be proposed below is not very different from existing procedures in which two or more tests are conducted and the results weighted according to their importance, in order to give an overall summary of level of performance.

- There is more emphasis on speed being a very important condition that varies from one accident to another.
- There is more emphasis on the possibility of using theory to estimate what test results would be if test conditions (in particular, speed) changed.
- Two components to importance are represented separately. One is the relative frequency of the condition (or combination of conditions) among accidents. The second is the cost (in particular, the likely severity) of accidents in that condition.

## *19.2 Proposed method: Measure-generalise-cost-average*

The average is typically the appropriate number on which to base a decision. This is the end point of a calculation as described below.

What is of chief interest in car impact tests is the injury. What is measured in an impact test is the acceleration pulse. This serves as a proxy for the injury. A contrast like this between what is of central interest and some measurable physical quantity is a feature of many other types of test. An acceleration pulse in an impact test is usually summarised by a single number: a calculation is carried out that results in the HIC.

Tests could be conducted at a number of different speeds. Alternatively, there may be some theory available concerning the dependence of HIC on speed (see section 4.5). Suppose that results are obtained, whether directly by testing or indirectly using theory or some other method, at lots of different speeds. It is often impossible to understand so many results, they need to be summarised. That is, an average needs to be calculated. Thus the results need to be numbers (and not words such as Good or Unsatisfactory).

It is likely that decisions are taken based on the set of test results. For example, whether the car has passed or not, or whether this car is better than that car. Therefore the number associated with a single test needs to be such that when several of these numbers are obtained, a decision can be made on the basis of the average. That is, they need to be "utilities" or "values" or "costs".

The proposed calculations are as follows.

- *Measure*. From the acceleration pulse obtained at one speed (or under one set of conditions), the HIC is calculated.
- *Generalise*. Either the test and HIC measurement are repeated for several speeds, or the results for various speeds are found using theory.
- *Cost*. Each HIC is converted to the corresponding cost.
- *Average*. From the costs at several speeds (or in several sets of conditions) together with the probabilities of these speeds occurring in the real world, the average cost is calculated.

A degree of dissatisfaction with the limitations of a single set of test conditions can be detected in the road safety literature from decades ago, and I hope this method has properly taken into account the contributions of such authors as Horsch (1987), Kanianthra et al. (1993), Korner (1989), Searle et al. (1978), and Viano (1988).

## *19.3 Equation for average cost*

Mathematical notation is used (section 19.3.2), but the meaning is spelt out in words.

### 19.3.1 Notation

Notation is given below.

- x is the speed of impact of the car with the pedestrian (assumed to be the same as that with which the head hits the car); more generally, this is any quantity that is specified for a test but varies in the real world.
- H(x) is HIC, the Head Injury Criterion; more generally, this is the outcome of the test.

- p(H) is the cost or utility associated with the test outcome H (the clinical nature of injury and the outcome vary randomly between different people even if HIC is the same, and in that sense p is an average); this is the quantity on which decisions are based.
- f(x) is the probability density function of x.

The cost p may be a true average dollar amount, including sums for the "value of life" and for pain and suffering. However, in the present state of knowledge, p is likely to be something simpler, such as the probability of death at a specified value of H.

Functions p(H) and f(x) are difficult to determine empirically, but estimates have been published, and were used by Hutchinson et al. (2012). For example, the probability of death as a function of HIC is included in Figure IV-10 of NHTSA (1995). A possible method of determining p(H) is to subject an instrumented headform and a dead human head to the same impact conditions; HIC is obtained from the headform, and physical damage (and hence p) is obtained by expert examination of the dead human head and expert assessment of the effects there would be in life of the injuries observed.

### 19.3.2 Average

The test takes place at some particular speed (e.g., 40 km/h), and H is observed. That implies some function H(x) at other speeds. The average cost is then given by the following integration.

$$Av(p) = \int p(H(x)).f(x).dx$$

The following puts this equation into words.

- Consider all conditions (in this example, all speeds of impact, x).
- Assume that the relative frequency of each condition is known. (f(x) specifies the relative frequency.)
- From a test result in one condition and a theory, work out what the test result would be in all conditions. (H(x) is this function.)
- Then convert each of these to a cost (that is, a number representing how bad it is). (p(H) is this function.)
- Use the frequencies of the conditions to average these costs.

Locations on the car differ in how safe or unsafe they are, and this equation refers to any particular location on the car. However, it may be desired to average over the whole car. Indeed, there is an equation that does that, very similar in principle to the equation above, in Hamacher et al. (2011). Furthermore, pedestrians vary in their head mass and stature, and these may affect injury. Thus x may not be a single quantity but instead a vector of quantities such as speed of impact, effective mass of the pedestrian's head, stature of the pedestrian, and so on. Stature of pedestrian may affect what location on the vehicle is struck. Details of elaborating the basic method in these ways are in Hutchinson et al. (2013) and in chapter 21 of Hutchinson (2018).

The equation given above economises on the number of tests by substituting a theoretical function H(x) based on one test result in place of empirical observations. See chapter 4 for such a theory.

Table 19.1. Example of use of the averaging equation. A discrete distribution of x is employed here. The probabilities are denoted f. There is an observed value of H at x = 40; from that, values of H at x = 20 and x = 60 have been calculated using some theory. For each H, there is a cost p.

| 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| f | x | H | H | p | p.f |
| | | | | | |
| 0.5 | 20 | | 88 | 10 | 5 |
| 0.3 | 40 | 500 | | 100 | 30 |
| 0.2 | 60 | | 1378 | 500 | 100 |

### 19.3.3 Example of calculation

Table 19.1 is an example of the mechanics of calculation.

- Instead of a continuous probability density function for f(x), there is a discrete distribution over three categories. It is supposed that three values of x (in column 2) occur with probabilities (column 1) 0.5, 0.3, and 0.2, respectively.
- Column 3 of the table shows the single value of H that was observed experimentally, at x = 40.
- Column 4 shows the values of H determined theoretically. Here, we choose x to be v and H to be HIC as given by the proportionality relationship of section 4.5, and assume that the exponent n = 1.
- Column 5 shows the costs (disutilities) associated with the respective values of H (indicative round numbers here, but determined empirically in a real calculation).
- Finally, column 6 shows the products p.f, the total of which is Av(p) = 135.

This is only a demonstration of how the calculations are done, and the units of x and p are not stated, as they are not relevant for such a demonstration. However, 40 km/h is a common speed in pedestrian headform impact testing (see chapter 2). Note that if chapter 4 were really to be linked with pedestrian impact testing, the angle of impact would be taken into account. In general-purpose testing, impact is usually normal to the surface (that is, at a right angle, 90 degrees). But that is not so for pedestrian headform testing. If the speed of impact is x and the angle between the direction of impact and the surface is θ, it would usually be assumed that this is equivalent to a normal impact at speed x.sin(θ).

## *19.4 Discussion*

### 19.4.1 Possible interaction of design and speed

A test protocol specifies a speed at which the test shall be conducted. Choice of this speed presumably takes into account both the many low-speed crashes and the high-speed crashes that are fewer in number but carry a much higher risk of death or serious injury. The test result indicates level of risk.

Another purpose of the test is to permit comparison of one model of car with another. Even if a test procedure were inaccurate as regards absolute level of risk, it might nevertheless be very useful if it provided a fair method of comparing different models of car. Thus the question arises, if one model of car performs better than another in the test, does it also perform better in a similar test conducted at a lower speed, and in a similar test conducted at a higher speed? In statistics and many other fields, lack of consistency in this respect would be referred to as "interaction" between car design and speed in their effects on test performance (protectiveness).

Such interaction is plausible. Suppose that car model A gives rise to a lower HIC than car model B at the standard test speed, but the bonnet of A is close to bottoming out whereas there is spare space available for further deformation under the bonnet of B. Then at a higher test speed, model A is likely to be much worse than at the standard speed, whereas model B will be only a little worse, and model A may now give rise to the higher HIC.

In principle, the averaging equation permits a relatively poor low-speed performance that worsens only slightly with increasing speed to be balanced against a relatively good low-speed performance that unfortunately worsens sharply with increasing speed. But there are substantial practical difficulties: the function H(x) is very poorly understood in the case of bottoming out, and the function p(H) is very poorly known in the case of the very high values of H that occur.

The above paragraphs are about differing strengths of the effect of speed. A slightly different issue is that of optimal stiffness when there are several speeds of impact. There then needs to be some sort of process of averaging over different speeds, and the relevant quantity needs to be one that can be averaged, as p can. See Appendix 6 for this.

### 19.4.2 Application to integrated assessment of primary and secondary safety

Probabilities f(x) specify how common are bad conditions, and the function H(x) specifies the effect of bad conditions on the object under test. Both f and H may change. Improvements to braking systems or tyres, and new technologies such as autonomous braking, may prevent some accidents and substantially reduce the impact speeds of others: there would be a change in the distribution of speeds, f(x). This constitutes an improvement of primary safety. Change to the design of the vehicle bonnet and to the stiff structures underneath are what may change the impact test result and the function H(x). An improvement in this respect refers to secondary safety.

The averaging equation depends on both f(x) and H(x), and thus permits the integrated assessment of both primary and secondary safety features. (There is no suggestion that secondary safety requirements should be relaxed for cars with good primary safety. Rather, it is envisaged that when improved primary safety becomes common in new cars, cars that lack those features should be subject to tightened secondary safety requirements.)

As an example of integrated assessment, consider Table 19.1 again. But now suppose that when x is 40, H is 550, which is a little worse. Corresponding to x = (20, 40, 60), H is predicted to be (97, 550, 1516). The costs p might be (12, 120, 600). Now suppose that the probabilities f change also, and are now (0.7, 0.2, 0.1). The sum of the three values of the product p.f is 8.4 + 24 + 60 = 92.4. That is lower than the total of 135 in Table 19.1: the change in the distribution of x has (in this example) more than compensated for the increase in H.

### 19.4.3 Equivalence between impact speed and test result

From the proportionality relationship in section 4.5, the ratio of benefits from a one per cent reduction in impact speed to those from a one per cent reduction in the HIC observed in a test is (4.n + 1)/(n + 1). This is between one and four, and is 2.5 if n is 1.

### 19.4.4 Frailty

The frailty of the person struck is important in determining the outcome. It is not represented in the equation of section 19.3.2: frailty is typically seen as outside the scope of the testing context, as a process of averaging over people occurs in the construction of the p(H) function. (I am using frailty in quite a broad sense to refer not only to weakness but also to other reasons for poor outcome from a given physical input.)

If, on the other hand, it were thought that the distribution of impact speeds f(x) were different for people of different frailties (of different ages, for example), then it would be necessary or desirable to represent frailty explicitly in the equation. There is further discussion in Hutchinson et al. (2013) and section 21.3.2 of Hutchinson (2018).

### 19.4.5 Similar calculations (1): Penetrating injury

The equation in section 19.3.2 might be relevant to many fields of testing, if there is appropriate interpretation of the functions H(.), p(.), and f(.). Two somewhat similar sets of calculations will be summarised here and in section 19.4.6.

Chapters 15 and 16 were about penetrating injury, and so is this example. It concerns inferring from a small number of tests of a specified missile (into a tissue simulant) what the average effect would be on the human body. The description of this in the report by Kokinakis and Sperrazza (1965) appears very different from the equation in section 19.3.2, but the core similarity has been noted by Hutchinson (2015). The interpretation there is that H(x) is the wound received (the name of the injury, or a description) when someone is struck at location x (the name of the part of the body), p(H) is the level of incapacitation (a percentage), f(x) is the probability of x being the part of body that is hit, and the result of the equation is the average level of incapacitation.

As with the head impact context, there is difficulty in establishing a credible function p(H). Kokinakis and Sperrazza refer to using medical assessors who estimate physiological effects in soldiers subjected to many hypothetical wounds, and to establishing a consensus of medical assessors and combat personnel on the percent disability (for a given tactical situation at a given time after wounding).

There is continuing interest in this. VanAmburg (2011) estimated the likely level of injury from a fragment at different impact points on the limbs or face. The calculation was then taken a step further, to give a probability over a random point of impact of a specified level of injury. In the present terminology, p(H(x)) is 1 if the AIS (Abbreviated Injury Scale) score corresponding to H(x) is at least 3, and is 0 otherwise, and the probabilities of different impact locations f(x) are assumed equal.

Tan et al. (2017) had software that was able to calculate various outcomes from a projectile (e.g., bullet or fragment) hitting a head. The calculations were performed for thousands of different trajectories of the projectile. The head was represented as wearing one of several helmets. One of the calculated outputs was head-to-helmet spacing, which might be positive (space maintained between head and helmet) or negative (impact of helmet with head, implying injury of severity related to the implied deformation of the head). The distributions of this quantity for different helmets were illustrated by Tan et al. The characteristics of the helmet (including how much of the head it covers, and whether it has padding) obviously play a big part in determining the results of the calculations. Although testing is, no doubt, important in establishing some of those helmet characteristics, testing is not so central in this context as it is in sections 19.2 and 19.3. And Tan et al. do not convert the calculated outputs to costs and average them.

The conversion of H to p has been represented in section 19.3 as having only one stage. However, several distinct stages may be identified. A particular named injury (e.g., skull fracture) might be likely at a given level of HIC, that injury might have a particular outcome (including a certain number of days in hospital, and a certain number of days off work), and that outcome might have a certain disutility (including economic losses and a monetary equivalent of pain and suffering). In section 19.3.2, p(H) was not split into several stages. However, something similar is done in the vulnerability / lethality context: "target damaged components" lead to "target measures-of-capability" which in turn lead to "target measures-of-effectiveness (utility)" (Deitz, 1998).

### 19.4.6 Similar calculations (2): Manufactured items

There was some consideration of damage of manufactured items in chapter 17. In that context, the hostility of the environment of a packaged item is of interest. This might be summarised by, for example, the distribution of heights from which a package might fall or be dropped (Peache, 1986, Figure 4). It is thought that when being handled by humans, small packages tend to be dropped from greater heights than large packages, i.e., the mean of the distribution is greater. A distribution of drop heights might be considered analogous to a distribution of impact speeds in section 19.3.

A procedure suggested by Burkhard (1966, pp. 169-170) is worth mentioning. The context was shock and damage to hearing aid transducers. I am unable to understand exactly what Burkhard meant. The following is my interpretation of what he intended to say.

- Each transducer has a fixed tolerance for acceleration. Failure occurs if tolerance is exceeded. Tolerance varies between specimens of the transducer.
- In what follows, the notation (symbols x, p, and f) will be chosen so as to emphasise similarity with section 19.3.2.
- Let p(x) be the cumulative distribution of tolerance. That is, the proportion of transducers with tolerance less than x is p(x).
- Consider a period of time, e.g., one year, and the maximum acceleration a transducer is exposed to in that year. Let f(x) be the probability density function of the maximum acceleration.
- For any x, p(x).f(x).dx is the proportion of transducers that experience x as the maximum acceleration in the year and have a tolerance less than that, and thus fail. (I should say: p(x).f(x).dx is the proportion of transducers that experience maximum acceleration between x and x + dx, the increment dx being small, and have tolerance less than that, and so on.)
- The area under the curve representing the product p(x).f(x) is the integral $\int$ p(x).f(x).dx. As Burkhard says, this is proportional to the number of transducers that fail in the relevant time period.
- In Burkhard's Figure 3, curve 1 is an example of p(x), curves 2 and 3 are examples of f(x), and curves 4 and 5 are examples of the product p(x).f(x).

The results of testing enter via observing sufficient tests (fail vs. tolerate an acceleration) to construct the cumulative distribution of tolerance p(x). This is a little different from observing a test result (for example, H(40)) and extending that into a function H(x). Burkhard had in mind a sensitive component being packaged within a case in order to be used, and Sheehan (1988) describes the above method in the context of packages being transported.

This might be described as a stress-strength model. A name like that, though, tends to suggest similarity and symmetry between stress and strength. But strength is observed by testing in a laboratory, whereas stress is estimated by in some way taking a sample from the environment; even if there is no positive reason to doubt the correctness of both measurements, the methods are likely to be different, which raises the question of comparability. My feeling is that to identify a situation as being analogous to a stress-strength model does not solve the problem; rather, it is like the equation in section 19.3.2 in that it is the first step on quite a long road towards solving the problem.

# 20. Concluding comments

There was a partial summing-up in sections 7.13 and 7.14, only about a third of the way through this book. At the stage reached now, many more examples have been given, and this chapter is able to add more comments of the same general type, about the treatment of data, about theories, and about experimentation.

As I said at the beginning of chapter 6, I am interested in data. Having a theory, or some alternative theories, about what is going on is usually helpful in understanding data.

I especially want to apply theory to impact test data because people are killed and injured in road accidents in many different circumstances (that is, at many different speeds, and so on). Without theory, I do not see how it can be possible to make full use of test data. In chapter 19, I have indicated how such knowledge might be applied. Perhaps the procedure there is too elaborate and data-intensive, but some rough form of it is likely to be practicable.

## *20.1 Suggestions about data analysis*

The most important point to come out of chapters 7 - 18 may simply a straightforward and conventional one about data description. That is, in view of what theories are available, it seems sensible to plot output variables against input variables on the basis that the relationships may be power funtions. So plot the logarithm of one variable against the logarithm of the other. When that is done, it can be asked whether the relationship is approximately a straight line, and what the slope is.

As described in section 6.1, it is worthwhile investigating whether the relationship between each of several independent (input) variables and each of several dependent (output) variables is a power function. When studying head impact, the most important ones are usually the following.

- Input variables: speed of impact, mass of headform, surface impacted.
- Output variables: maximum deformation, maximum acceleration, HIC.

Having considered separately each of several pairs of variables and estimated the exponents, the next step is to ask whether all the exponents are consistent with some particular value of n.

An important alternative theory is that exponent n is 1 (or perhaps 1.5, or perhaps 0), with any deviations from that being due to extra phenomena that occur in a limited range of circumstances, or which are not really the subject of the experiment under discussion. Phenomena at initial contact, and bottoming out, are two important cases.

- If n is thought to be 1, a different method of plotting will be used. If something extra happens at initial contact, it is likely to mean that the intercept will be non-zero. A scale for the scatterplot should be chosen so that can be examined. It may be possible to conduct another experiment specifically to study the size of the extra phenomenon or distortion of results.
- Bottoming out is perhaps more important, as injury can be very severe. Understanding of the relevant equations and knowledge of how frequently it occurs are both often quite poor. As will be said in section 20.5, deformation and the available deformation distance should be reported. That will enable an estimate to be made of the impact speed (and other conditions) at which bottoming out will occur and theory will fail.

I might add that in a sense, the idea that n is 1 but extra phenomena occur is not an alternative to the idea that n might be anything. Rather, the two ideas could be combined. But the problem is that with most datasets, three parameters is too many. (One parameter would be needed to reflect the size or importance of the special phenomenon at initial contact, and two would reflect the main phase of the impact, and might be analogous to the slope and intercept after taking logarithms.)

I think it is fairly clear that approaching data description and analysis in this way will be helpful. It will lead to questions such as whether discrepancies between data and theory are real and worth studying. It is not clear to me what the way forward from there will be; in particular, it is not clear that experimentation will be able to become more extensive and more precise, in order that such questions can be answered.

It is possible that one or general characteristics of the shape of the acceleration pulse might be worth studying. The characteristics might include: whether the pulse is single-peaked or has multiple peaks, whether the peak is early in the pulse or late, and whether the peak is narrow or broad relative to the total duration.

## *20.2 Suggestions about theories*

Theories have already been mentioned in section 20.1.

The major theory is the Hunt and Crossley differential equation (section 4.3), which leads to proportionality relationships between input and output variables (section 4.5).

Some alternatives to this, or perhaps variations on the idea, are as follows.

- Assume that n = 1, with any apparent deviations from that being due to something extra going on (see section 5.4).
- Assume that n = 1.5, with any apparent deviations from that being due to something extra going on.
- Assume that n = 0, with any apparent deviations from that being due to something extra going on.
- There might be some specific reason why the proportionality relationships apply only to a subset of the inputs, or a subset of the outputs, or a subset of the combinations.

If bottoming out is of specific concern, there is a theory and method of plotting data available from packaging engineering (see section 17.4). However, this refers to gradual bottoming out, not to sudden bottoming out as when the underside of a car's bonnet hits something much stiffer.

## *20.3 Suggestions about the reasons for theories*

My opinion is that successful data description or prediction by a theory is only one of its purposes.

The implications of a dataset for a theory may be not clear, with some aspects supported and others apparently contradicted. But even if the theory seems to be neither decisively supported nor decisively contradicted, it may generate specific questions that open a new path to better understanding. It provides a base on which people can think about and talk about the subject, it is something that people can criticise and attempt to improve.

The results in this book demonstrate that it is now practicable to draw conclusions from comparing theory with data. However, most datasets do not have enough data points to give us confidence about the degree of success of the theory, or to estimate n accurately.

Theory may suggest oddities in the data, as when slopes of relationships are incompatible with the predictions in section 4.5. The apparent suggestion is that the theory is wrong. But often the possibility of the data being wrong should also be considered.

- For example, consider the slopes of the dependence of $\ln(A_{max})$ on $\ln(v)$ and the dependence of $\ln(S)$ on $\ln(v)$. In section 4.5.1, these are found to be $2.n/(n + 1)$ and $2/(n + 1)$. Consequently, their sum is 2: if one is big, the other is small, and if one is bigger than 1, the other is smaller than 1.
- Having found this for a specific theory, it may be thought that it is not so much a theoretical result, but more like common sense.

- And if data disagrees with common sense, it is desirable to carefully examine the possibility of there being some mistake.

Theory makes it possible to interpret and generalise experimental results. Without a theory, it is not clear what to do with data. With a theory, the tasks --- including estimation of the unknown parameters, and specification of the range of experimental conditions --- are much better defined.

## *20.4 Suggestions about experimentation*

It seems that the management of experimentation does not often live up to the ideal. The ideal I am thinking of is systematically varying an independent variable, observing the results, comparing them with a theory of what is happening or with a potentially useful model, designing another experiment that will confirm or disconfirm the theory (e.g., vary another independent variable, or observe another dependent variable), conducting that experiment, and so on.

It is natural for experimenters to give priority to the specific task at hand. But to the extent possible, they should remember that their results may be of relevance to a broad community concerned with cushioning impact. This community may, for example, have greater interest in what law-like relationships are obeyed than in the specific value of HIC at a specific impact speed.

The experimental set-up used needs to be simple and reproducible. Reproducibility leads to precision in the estimates of the results. In particular, improved precision may make it easier to estimate how high an impact speed will lead to breakdown of the equation: limited thickness of cushion is very common, which will eventually mean bottoming out and breakdown of the equation. I am inclined to think that experimenters are sometimes too concerned with realism; admittedly, it is difficult to distinguish between essentials and inessentials. I am not intending to suggest any departure from good experimental practice. More consistent experiments are often possible if things are left unchanged that ought to be re-set --- I am certainly not advocating artificial improvement of this kind.

The research literature makes it clear that experimenters have usually conducted very few tests. Conducting many tests rather than few would lead to improved precision of the estimates of the parameters of the theory (typically, the exponent and the constant of proportionality). That means many tests in all conditions that are individually of interest. For example, low energy impacts with no bottoming out and high energy impacts with bottoming out may both be of interest --- and it is likely these will obey different laws and will have to be treated separately.

It is desirable that more than one dependent variable be measured and reported. Many studies have used either $A_{max}$ or HIC as the only dependent variable. As a check on the results, it is better to use both. If n is similar whether based on $A_{max}$ or on HIC, this increases confidence in the results. If the n's based on the two variables are different, this is a warning that the theory may not be valid and should be rejected or modified. Or perhaps there is something wrong with the calculation of $A_{max}$ or HIC. Maximum deformation is another very important dependent variable.

It is desirable that more than one independent variable be used. Impact speed is usually the centre of interest, and headform mass also is important. I am surprised it is so rare for stiffness to be measured.

As well as those discussed in this book, I have looked at a lot of other reports and papers on impact tests. The ones that I have discussed have certainly not been singled out as worthy of criticism. Rather the opposite: they achieve some success in that I consider them of interest to myself and to readers of this book.

A lot of observations (probably covering a wide range of the independent variable(s)), several independent variables, several dependent variables --- that sounds as if the programme of experimentation will be very expensive.

- It is true that I suspect there is much less experimentation than there ought to be --- I said so in section 1.4.
- But my point at the moment is that a cost constraint ought to lead to careful planning, imagination of what data analyses there will be, and decisions about priorities.

Continuing from the previous paragraph, I am now thinking of impacts (of course), that x (the independent variable of chief interest) and y (the dependent variable of chief interest) are both quantities that cannot be negative, and that there is quite a strong positive dependence of y on x. They might be acceleration or force or distance, and velocity or mass or stiffness, for example. Here is a list of several things you might want to know.

- What y is, for some fairly typical value of x.
- How much y changes, from the lowest realistic value of x to the highest.
- Whether y changes linearly or nonlinearly as x changes.
- Whether y is zero or non-zero when x is zero; and whether for small but non-zero x, it might be the case that y is zero.
- Specifically, whether the idea that y is a power function of x is supported or rejected; and whether the idea that y is linearly dependent on x, with a non-zero intercept on one or other axis, is

supported or rejected. At several points in this book, data is discussed that is compatible with both these hypotheses.

Each will suggest different ways in which to design the experiment, concentrating on the chief question and rather neglecting the others. If a choice is not made (that is, if you attempt to answer all the questions), there will not be many opportunities to economise, and a great number of experiments will indeed be needed.

## *20.5 Suggestions about reporting of experiments*

As I said in section 18.2, in my opinion, much reporting of experimentation is not as useful as it might be. Information is missing, some details of the method are not clear, there seem to be misprints, some aspects of the results are omitted, possible anomalous observations are not discussed and perhaps omitted from the main data analysis, and so on.

Three specific points are as follows.

- Experimenters should be clear about what was measured and how, and what consequences the choice of method might have. (For example, was acceleration or force measured; and was it measured at the centre of gravity of the impactor, or at contact, or below the contact plate; and what are the implications of the filtering method adopted?)
- Bottoming out is very important. Therefore, the deformation and the available deformation distance should be reported.
- Experimenters should give their judgment about important matters that the reader will be interested in, even if they cannot prove their judgment is correct. (They are in a better position to express an opinion than anyone else.) This might include what happened, why it happened, what assumptions are reasonable, roughly how accurate an observation is, whether specific observations might be misleading, and so on.

## *20.6 Suggestions about the results discussed in this book*

There have been a lot of examples discussed in this book. I think in each case the new viewpoint that has been provided by the hypothesised proportionality relationships (section 4.5) has meant that I have said something that is worth saying.

Across the set of examples, I would say not many generalities of a positive nature have emerged. A generality of a rather negative nature is that theory is typically neither convincingly confirmed not convincingly disconfirmed. Hence the disappointing conclusion (section 7.13) that the quantity of experimentation is insufficient: not enough tests, not enough independent variables considered, not enough dependent variables considered, not a wide enough range of speed and other inputs.

Several predictions of the theory of chapter 4 were supported by the datasets examined. In particular, relationships were often quite good straight lines when the logarithms were plotted.

But the support should not be exaggerated. Standard errors are typically wide, because there are few data points. And arguments can be advanced against the theory --- for some datasets, rather different values of n were estimated from different pairs of variables, and some slopes fell outside the predicted range.

# 21. Appendix 1: Bottoming out

When a headform (or a pedestrian's head) hits some part of the front of a car, bottoming out refers to contact between the underside of the bonnet (or other surface structure) and some much stiffer structure beneath (such as the engine). This is potentially very serious: there may be a great increase in HIC, maximum acceleration, other proxies for injury, and likely severity of injury.

Another form of bottoming out is more gradual, as when more and more crushing of a cushioning foam occurs between two rigid surfaces.

- See section 17.4 for the case of force being proportional to 1/(t - x), where t refers to the thickness of the material being crushed.
- See section 4.2.4 for the case of force being proportional to tan(x*.(π/2)) (where x* is the ratio x/t).
- The case of force being proportional to a power function of x/(t - x) has received some attention in vehicle safety (e.g., Trella et al., 1991; Kanianthra et al., 1993).

As far as I know, attempts to model sudden bottoming out have been rare. The paragraphs below simply set out some things that would need to be considered.

Everything changes when bottoming out occurs. There is no reason to think that results from the bonnet alone deforming have any implications for what happens when bottoming out begins. (Of course, it is common sense that if the speed of the head has been substantially reduced when the very stiff structure is reached, the consequences are likely to be less severe than if the speed is high.)

The conditions (effective mass, impact speed) that lead to bottoming out can be predicted.

- Measure the distance available for deformation.
- Conduct an impact test of the relevant type, measuring the deformation at some known speed.
- Use the formula in section 4.5 to predict the speed at which available distance will be exceeded. (Some assumption about the value of the exponent n will be neccessary.)

Extrapolation like this is not exact, but reasonable accuracy is likely. Conditions for a pedestrian's head bottoming out are likely to be the same as for a headform, as the headform's mass has been chosen appropriately.

It is difficult to imagine preventing bottoming out at high speeds. But it is possible to imagine the wider use of crushable foams on the undersurface of the bonnet, and the replacement of sudden bottoming out with a more gradual bottoming out as the foam crushes more and more.

# 22. Appendix 2: Improvements in the design of cars for pedestrian impacts

It was mentioned in section 2.2 that Lawrence et al. (2006) demonstrated several methods for improving the pedestrian test performance of specific models of cars. Other papers of that type were also cited in section 2.2. This Appendix is based on part of Hutchinson et al. (2011).

Since 1997, the impact laboratory at the Centre for Automotive Safety Research, University of Adelaide, has conducted pedestrian headform and legform testing on behalf of ANCAP, plus tests for other clients and other purposes. Several improvements have been noted over the years.

*Beneath the bonnet*. One of the most significant has been the lowering of under-bonnet components, in particular, battery terminals, suspension strut mounts, and engine components. This has been particularly evident in Japanese-manufactured vehicles.

*The side and hinges*. Another significant change that has been noticed is collapsing side guards. In the past, sufficient strength and stiffness of the bonnet and guards to support the weight of people leaning on the vehicle was achieved through thickness of material and rigid support. Using better selection of materials, some manufacturers have reduced the thickness of the panelwork, reducing weight while keeping strength. Attachment of the side guards to stiff reinforced sections of the vehicle is being replaced by the use of supports or brackets that collapse or deform under impact. Typically, attachment of the bonnet to the hinges has been difficult to design safely. Years ago, testing on hinges often resulted in puncturing of the hinge screws through the bonnet top, and extremely high accelerations and HIC have been recorded. Various alternative methods of attachment have been trialled with varying levels of success in lowering HIC and acceleration, but no method is yet considered particularly safe.

*The rear of the bonnet*. Wiper pivots have typically been very injurious. To combat this, some manufacturers have redesigned the wiper pivot on a frangible assembly. A change in design of the firewall has also been noted in various vehicles, where the stiff top section of the firewall that had supported the rear of the bonnet has been lowered or offset rearward to the base of the windscreen. The bonnet has then been supported by a collapsible plastic plenum. Support at the base of the windscreen has and continues to be a notable injurious area. However, a few examples have been seen of this support providing adequate support for the windscreen but yielding when impacted, giving a passable HIC result.

*The bonnet itself*. Its own rigidity and strength requirements can lead the bonnet itself to be injurious, particularly the reinforcement or ribbing and at the extreme front. Some new bonnet designs have seemingly consistent stiffness characteristics across the entire bonnet, the underside reinforced sections having the same impact characteristics as the rest of the bonnet.

# 23. Appendix 3: Is use of an acceleration-based proxy for injury evidence that acceleration is the most important factor?

The use of acceleration-based proxies for injury, such as HIC and $A_{max}$, is often specified in testing. In section 3.3.2, the question was raised whether this constitutes evidence that the experts who wrote the test protocols believed that acceleration (rather than force or deformation or something else) is the most important factor in injury causation.

As a preliminary, it seems very likely that the expert committees took into account a variety of factors, including (importantly) the practicability of measurement, when specifying a proxy for injury. It is unlikely they decided solely on the basis of injury causation.

## *23.1 A limited aim for a proxy measure*

In using a proxy measure for injury, some people may have quite a limited aim, while others may have a broader one.

The intention is that in tests of different things, each test being in the same specified conditions (such as using specified equipment and being at a specified impact speed), the results are in the same order as injury severity would be. This is what I mean by a limited aim, or a limited area of application for the proxy measure.

In the case of pedestrian headform tests (chapter 2), the "different things" are the different locations on different vehicles, or (when the results have been aggregated to the level of the vehicle) the different vehicles themselves.

The "same order" means that if injury would be worse for impact location 1 than for impact location 2, the numerical result in the test is worse for impact location 1 than for impact location 2.

## *23.2 A broader aim for a proxy measure*

Tests are conducted in defined conditions. For example, the impact speed is specified, and so is the headform mass. It was said above that

(1) The intention is that in tests of different things, the results are in the same order as injury severity.

Notice that that does not require (for example) that

(2) In tests at different speeds, the results are in the same order as injury severity,

or that

(3) In tests of different things at different speeds, the results are in the same order as injury severity.

Statement (2) above is not really a concern --- HIC (the Head Injury Criterion) and $A_{max}$ (maximum acceleration) both obviously increase with impact speed.

But statement (3) certainly is. It is possible for a proxy for injury to reflect correctly the object being tested, and to reflect correctly the speed of impact, but not to reflect correctly the combination of object and speed.

The following is a simple example. Consider two objects A and B, tested at two speeds, 10 and 15. Injury severity is not usually available, but in this demonstration it is supposed to be as shown below, and the proxy for injury is also given.

A, 10, injury = 0, proxy = 0.
A, 15, injury = 1, proxy = 2.
B, 10, injury = 2, proxy = 1.
B, 15, injury = 3, proxy = 3.

The proxy measure behaves correctly as regards the effect of object (there is the same ordering as for injury, at both speeds), and behaves correctly as regards the effect of speed (there is the same ordering as for injury, for both objects). However, it does not correctly reflect the ordering of the four combinations of object and speed.

Such data as this is not usually thought about. For one thing, injury is not often studied directly in experiments. For another, tests are usually at a single specified speed. The point here is to illustrate that different people may have different standards for the validity of a proxy for injury. Some may insist on (3) above; for other people, (1) and (2) may be sufficient.

In addition, in tests of different things at different speeds, HIC may be in a different order to $A_{max}$ (see section 6.5.4). Thus they cannot both be in the same order as injury severity. At least one of them must be inadequate in this respect.

## *23.3 What do most people mean most of the time?*

I think that most people who claim that HIC (for example) is suitable for use as an indicator of likely head injury severity have the broader interpretation of HIC in mind. That is, they consider that in the range of circumstances in which it is likely to be used (that is, various objects tested, at various speeds, and perhaps with headforms of different masses), HIC will be in the same order as injury severity. Therefore I consider that interpretation can be adopted. But we should also bear in mind that it is possible for (1) to be correct but not (3).

The fact that an expert committee decides that the acceleration pulse is what should be recorded in an impact test, and that HIC should be used to summarise it, is evidence that the acceleration pulse and HIC are suitable for that purpose. However, it is not conclusive evidence that the expert committee believe in (3) --- belief in (1) would probably have been sufficient reason for the decision.

Statement (1) is compatible with thinking (for example) that what matters is something other than acceleration (e.g., deformation of the head), but when comparing vehicles, this other quantity correlates highly with headform acceleration. Chapter 3 makes the point that some types of injury may be related to acceleration of the human and other types of injury may be related to deformation of the human. Hence the possibility must be mentioned that acceleration is not important in itself but as an indicator of human deformation. (Section 5.2.2 finds a close theoretical connexion between human deformation and acceleration.) It is convenient to proceed as if there is a consensus in favour of (3), but that may not really be so.

Various people have conducted experiments in which some aspect of the impact is changed, and the effect on HIC is observed. Experimenters who change impact speed (for example) presumably believe that HIC remains valid as an indicator of injury when impact speed changes.

# 24. Appendix 4: Effect of random variation on the estimate of n

In section 7.13, it was noted that in impact experiments the independent variable often has only a limited range, the slope of the scatterplot is estimated only imprecisely, and the exponent n is estimated only imprecisely. This Appendix will give more details of that argument. Both Appendix 4 and Appendix 5 might be considered to come within the topic of propagation of error.

The following refers to $A_{max}$, but similar calculations could be made for other dependent variables such as HIC and S.

Suppose we conduct experiments at some impact speed and at twice that speed, and that the observed values of $A_{max}$ are $A_1$ and $A_2$. The exponent describing the dependence of $A_{max}$ on v will be estimated as $\ln(A_2/A_1)/\ln(2)$, and this will be interpreted as $2.n/(n + 1)$. If the exponent describing the dependence of $A_{max}$ on v is $p_A$, n will be $p_A/(2 - p_A)$.

Now suppose $A_{max}$ at the higher speed is $(1 + d).A_2$, instead of $A_2$, where the fractional difference d reflects random variation, measurement error, and so on. How much effect will this have?

As an example, suppose that n is 1, and so the ratio $A_2/A_1$ is 2 if d is 0. When d is non-zero, $\ln((1 + d).A_2/A_1)$ is $\ln(1 + d) + \ln(2)$ instead of $\ln(2)$. Thus $\ln((1 + d).A_2/A_1)/\ln(2)$ is $1 + (\ln(1 + d)/\ln(2))$ instead of 1.

The fractional error d is likely to be quite small. An elementary property of the natural logarithm is that $\ln(1 + x)$ is approximately x when x is small. Thus $1 + (\ln(1 + d)/\ln(2))$ is approximately $1 + 1.44 \times d$.
If d is 0.1, for example, the result is 1.144. That is, a 10 per cent error in one observation has led to a 14 per cent error in the estimated slope.

Further, the exponent n will be $1.144/(2 - 1.144)$, which is 1.34. That is, a 10 per cent error in one observation of $A_{max}$ has led to a 34 per cent error in the estimated n.

This is worth knowing, but it is questionable whether it should be taken at face value. The meaning of a certain size of error lies largely in its importance, and I am not sure that a given percentage error in an observation of $A_{max}$ is roughly as important as the same percentage error in n.

# 25. Appendix 5: Comparison of three estimates of n

## *25.1 Notation, and elementary relationships*

Experiments are often performed in which impact speed v is varied, and maximum acceleration $A_{max}$ and the Head Injury Criterion HIC are measured.

There are then three estimates of the exponent n: from the dependence of $A_{max}$ on v, from the dependence of HIC on v, and from the dependence of HIC on $A_{max}$. Let these be nAv, nHv, and nHA.

It might be thought that as v has been experimentally manipulated, nAv and nHv will be used, and there will be no reason to given consideration to nHA. However, nHA has the advantage that it does not rely on the measurement of v being valid and reasonably accurate.

In what follows, I am not thinking of random variation of a single data point from a relationship, but of n that is inferred from the relationship. Both Appendix 4 and Appendix 5 might be considered to come within the topic of propagation of error.

Suppose that the true value of the exponent is n, and that $d_A$, $d_H$, and $d_{HA}$ are the fractional differences of nAv, nHv, and nHA from n. (The d's may be positive or negative.) That is,

nAv = $(1 + d_A).n$

nHv = $(1 + d_H).n$

nHA = $(1 + d_{HA}).n$

The question at issue in this Appendix is whether, if $d_A$ and $d_H$ are small, $d_{HA}$ is necessarily small also. The answer is no. That is, it can happen that there is little difference between nAv and nHv, but nHA is very different from both.

Let $p_A$ be the exponent when plotting $A_{max}$ versus v. Similarly, $p_H$ is the exponent when plotting HIC versus v, and $p_{HA}$ is the exponent when plotting HIC versus $A_{max}$.

The estimates nAv, nHv, and nHA are calculated from the known quantities $p_A$, $p_H$, and $p_{HA}$ using the relationships below (obtained in section 4.5).

$p_A = 2.nAv/(nAv + 1)$

$p_H = (4.nHv + 1)/(nHv + 1)$

$p_{HA} = (4.nHA + 1)/(2.nHA)$

And the p's satisfy the following relationship.

$p_{HA} = p_H/p_A$

## *25.2 Relationship between the fractional differences*

Because $p_{HA} = p_H/p_A$, there is the following relationship between $d_{HA}$ (on the one hand) and $d_A$ and $d_H$ (on the other).

$[4.n.(1 + d_{HA}) + 1] / [2.n.(1 + d_{HA})]$

$= \{[4.n.(1 + d_H) + 1] / [n.(1 + d_H) + 1]\} \times \{[n.(1 + d_A) + 1] / [2.n.(1 + d_A)]\}$

After some algebra (I hope I have not made a mistake), the following relationship is found.

$d_{HA} = [(1 + 4.n).d_A - 3.n.d_H + n.d_A.d_H] / [1 + n - 3.n.d_A + 4.n.d_H]$

## *25.3 Consequences*

The expression for $d_{HA}$ just obtained appears unlikely to be a problem: if $d_A$ and $d_H$ are close to zero, the numerator will be close to zero and the denominator will be close to 1 + n, and so $d_{HA}$ will be close to zero. Thus nAv, nHv, and nHA will all be close to the true value n.

Unfortunately, that is misleading. The number of impact tests is usually small, and impact tests are often quite variable in their results. Consequently, the estimates nAv and nHv have quite large standard errors associated with them. If they were 1.2 and 0.8, they might be regarded as consistent with each other, and as suggesting that n is about 1. In other words, "small" might mean that the magnitude of $d_A$ and $d_H$ is about 0.2.

Thus suppose that n = 1, $d_A = 0.2$, and $d_H = -0.2$. Using the above formula, it is found that $d_{HA}$ is 2.6. That is, nHA is 3.6, even though nAv and nHv are respectively 1.2 and 0.8.

If $d_A$ is negative and $d_H$ is positive, the discrepancy is in the other direction. Suppose that n = 1, $d_A$ = -0.2, and $d_H$ = 0.2. It is found that $d_{HA}$ is -0.48. That is, nHA is 0.52, even though nAv and nHv are respectively 0.8 and 1.2.

# 26. Appendix 6: Stiffness when there are several speeds of impact

## *26.1 Optimum stiffness*

Choice of stiffness of a cushion is a complex matter, and I acknowledge that I am only giving one argument.

The example here is a car's bonnet struck by a pedestrian headform. The clearance distance available under the bonnet before bottoming out occurs is presumed to be specified.

At any speed, there is an optimum stiffness of the bonnet. It is approximately the stiffness such that the clearance distance is exactly used up in stopping the headform (see sections 2.1 and 2.3). Increase the stiffness above this, and injury severity increases: the bonnet is unnecessarily stiff for the speed being considered. Decrease the stiffness below the optimum, and injury severity increases sharply: the bonnet is not stiff enough to prevent bottoming out, and this occurs to a greater and greater extent as stiffness decreases.

Optimum stiffness depends on the impact speed. Stiffness corresponding to minimum injury is greater for a high speed of impact than for a lower speed of impact. Chapter 19 gives a method of calculating an estimate of safety performance averaged over the range of speeds that do occur, and section 19.4.1 referred to optimal stiffness over a range of speeds.

Suppose the population of speeds of impact is represented by two speeds of equal frequency of occurrence. For each speed, injury severity depends on stiffness in the way described. Consider injury severity averaged over the two speeds. For what stiffness is average injury severity minimised, that is, what is the optimum stiffness?

I think the optimum stiffness is only a little lower than it is for the higher speed. The reason is the unequal slopes of the two relevant functions. That is, below the stiffness that is optimum for the higher speed, severity at the lower speed decreases with decreasing stiffness, but severity at the higher speed increases sharply with decreasing stiffness because bottoming out is occurring. Consequently, the average increases with decreasing stiffness.

This argument is not rigorous.

- Injury severity needs to mean roughly what it usually means, of course. In addition, the argument relies on injury severity being defined in such a way that averaging is valid. With that restriction, it is not clear that the dependence of injury severity on stiffness really is much steeper when bottoming out occurs than when it does not.

- For many functions, the lowest point is flat. Thus for the higher speed of impact, the dependence of injury severity on stiffness may be weak in the vicinity of the optimum stiffness. The counter-argument to this is that over the range between the two optimum stiffnesses, there will be a greater change of injury severity in the case of the higher speed of impact (because of bottoming out), and thus the best choice of stiffness is likely to be closer to the optimum for the higher speed than to the optimum for the lower speed.

Nevertheless, I think it likely that it is roughly correct that the optimum stiffness is only a little lower than it is for the higher speed.

## *26.2 Representing a distribution of speeds by two speeds only*

What two speeds of equal frequency of occurrence best represent the population of speeds of impact?

- Let *mean* and *s.d.* be the mean and standard deviation of the population of speeds.
- The two speeds *mean* + *s.d.* and *mean* - *s.d.* are a suitable choice. This is because if a population of speeds consists of these two speeds in equal proportions, its mean and s.d. are respectively *mean* and *s.d.*

Consequently, the suggestion here is that for a given clearance distance, the optimal stiffness for the variety of speeds of real-world impacts is the optimal stiffness for speed *mean* + *s.d.*

## *26.3 Discussion*

I should warn readers that I think that people interested in this subject will consider that to be too high a speed and too high a stiffness.

I said at the beginning of this Appendix that this is a complex subject, and many other arguments might be made. I certainly do not imply that the argument made here should prevail without considering other arguments. Nevertheless, I think it quite possible that in choosing stiffness, inadequate weight is often given to bottoming out in high speed impacts.

Variability of speed has been considered here. Variability of effective head mass could be treated in the same way. Some categories of people have less mass than others: children, for example. Less mass means less deformation (for given stiffness and given impact speed). Stiffness can be lower before bottoming out occurs.

## Acknowledgements

Many people have contributed to knowledge of blunt injury and damage, and have published this knowledge. Many people have personally helped me, over a long period. I thank all of these.

# References

The section, sub-section, chapter (Ch), or Appendix (App) where each reference is cited is shown in square brackets.

Ahmad N, Taha Z, Ujihashi S, Tanaka K 2009. An experimental study of the impact of a sepak takraw ball on a flat surface. In: The Impact of Technology on Sport, III. Proceedings of the 4th Asia-Pacific Congress on Sports Technology, pp. 447-452. Melbourne: RMIT University Press. [8.8]

Ali M, Sun J, Lam N, Zhang L, Gad E 2014. Simple hand calculation method for estimating deflection generated by the low velocity impact of a solid object. Australian Journal of Structural Engineering, 15, 243-259. [5.4.3]

Alves M, Yu J 2005. Material influence on the response of impacted beams. Latin American Journal of Solids and Structures, 2, 167-193. [17.13]

Ames E, Martin P 2015. Pop-up hood pedestrian protection. Paper 15-0111, 24th Enhanced Safety of Vehicles Conference. [2.2]

Anderson GB, Carter TC, Morley Jones R 1970. Some factors affecting dynamic fracture of egg shells in battery cages. In: R. F. Gordon and B. M. Freeman (Editors), British Egg Marketing Board Symposium No. 6, pp. 52-69. Edinburgh: Oliver and Boyd. [7.14, 18.12]

Anderson PSL, LaCosse J, Pankow M 2016. Point of impact: The effect of size and speed on puncture mechanics. Interface Focus, 6, paper 20150111. (This has an online supplement.) [16.4]

Anderson RWG, Long AD, Serre T 2009. Phenomenological continuous contact-impact modelling for multibody simulations of pedestrian-vehicle contact interactions based on experimental data. Nonlinear Dynamics, 58, 199-208. [4.2.5]

Anderson RWG, White K, McLean AJ 2000. The development of a protective headband for car occupants. Report CR 193, Australian Transport Safety Bureau, Canberra. [7.4]

Antona-Makoshi J, Davidsson J, Ejima S, Ono K 2016. Development of a comprehensive injury criterion for moderate and mild traumatic brain injuries. International Journal of Automotive Engineering, 2016, 69-75 (paper 20164086). [3.3.4]

Arakawa K, Mada T, Komatsu H, Shimizu T, Satou M, Takehara K, Etoh G 2006. Dynamic deformation measurement of a golf ball during normal impact. (In Japanese.) Transactions of the Japan Society of Mechanical Engineers, Series C, 72, No. 724, 132-137. [8.8]

Ballistic Impact Research Laboratory 2011. Test methodologies for the assessment of less-lethal kinetic energy rounds. Report from the Ballistic Impact Research Laboratory, Bioengineering Center, Wayne State University, Detroit, MI. Report available from the

National Criminal Justice Reference Service, National Institute of Justice, Rockville, MD. [12.5.2]

Banerjee A, Chanda A, Das R 2017. Historical origin and recent development on normal directional impact models for rigid body contact simulation: A critical review. Archives of Computational Methods in Engineering, 24, 397-422. [4.2.5]

Barkan P, Sirkin MF 1963. Impact behavior of a nitrile elastomer. SPE (Society of Plastics Engineers) Transactions, 3, 210-219. [17.3.2, 17.4.4]

Bass CD, Davis M, Rafaels K, Rountree MS, Harris RM, Sanderson E, Andrefsky W, DiMarco G, Zielinski M 2005. A methodology for assessing blast protection in explosive ordnance disposal bomb suits. International Journal of Occupational Safety and Ergonomics, 11, 347-361. [12.6.1, 12.6.2, 12.6.3, 12.6.4]

Becker EB, Anishchenko DV, Palmer SB 2015. Motorcycle helmet impact response at various levels of severity for different Standard certifications. Proceedings of the IRCOBI Conference, held in Lyon. http://www.ircobi.org/wordpress/downloads/irc15/default.htm [2.3]

Ben-Dor G, Dubinsky A, Elperin T 2015. Analytical engineering models for predicting high speed penetration of hard projectiles into concrete shields: A review. International Journal of Damage Mechanics, 24, 76-94. [15.1, 15.7, 16.5.2]

Berry NK, Fielke JM, Saunders C 2014. Determination of impact energy to devitalise annual ryegrass (*Lolium rigidum*) seed from one impact using double and single sided impacts. Biosystems Engineering, 118, 138-146. [18.4]

Bonin SJ, Luck JF, Bass CR, Gardiner JC, Onar-Thomas A, Asfour SS, Siegmund GP 2017. Dynamic response and residual helmet liner crush using cadaver heads and Standard headforms. Annals of Biomedical Engineering, 45, 656-667. [7.11]

Breeze J, James GJ, Hepper AE 2013. Perforation of fragment simulating projectiles into goat skin and muscle. Journal of the Royal Army Medical Corps, 159, 84-89. [Ch15 (1st para.), 16.1.1, 16.1.2, 16.1.3, 16.1.5, 16.1.7]

Burgess GJ 1988. Some thermodynamic observations on the mechanical properties of cushions. Journal of Cellular Plastics, 24, 56-69. [17.4.2]

Burgess G 1990. Consolidation of cushion curves. Packaging Technology and Science, 3, 189-194. [17.3.1, 17.4.2]

Burgess G 1994. Generation of cushion curves from one shock pulse. Packaging Technology and Science, 7, 169-173. [17.3.1, 17.4.2]

Burkhard MD 1966. Protection against shock and vibration. Journal of the Audio Engineering Society, 14(1), 168-174. [17.6, 19.4.6]

Cámara Tobalina J, Calleja-González J, Martínez de Santos R, Fernández-Lopez JR, Arteaga-Ayarza A 2013. The effect of basketball footwear on the vertical ground reaction force during the landing phase of drop jumps. Revista de Psicología del Deporte, 22, 179-182. [14.8]

Carter TC 1970. Why do egg shells crack? World's Poultry Science Journal, 26, 549-561. [18.12]

Cerruto E, Aglieco C, Gottschalk K, Surdilovic J, Manetto G, Geyer M 2015. FEM analysis of effects of mechanical impact parameters on fruit characteristics. Agricultural Engineering International: CIGR Journal, 17, 430-440. [18.5, 18.10]

Chang L-T, Huang T-J 2007. Protective performance of plate-cell rubber tiles against childhood head injury on playground surfaces --- A finite element analysis. Journal of Solid Mechanics and Materials Engineering, 1, 842-853. [10.1, 10.6.1, 10.6.2]

Chang L-T, Tsai K-H, Shiau J-S 2004. Experimental evaluation of the impact-absorption properties of rubber tiles for the playground. Biomedical Engineering: Applications, Basis and Communications, 16, 244-250. [10.5]

Chen JY, Miyazato M, Ishiguro E, Nanba N 1993. Detection of firmness in intact pumpkins and Sakurajima radishes by impact forces. Journal of the Japanese Society for Horticultural Science, 61, 951-956. [18.1]

Chen P, Ruiz-Altisent M, Barreiro P 1996. Effect of impacting mass on firmness sensing of fruits. Transactions of the ASAE (American Society of Agricultural Engineers), 39, 1019-1023. [18.9.1]

Cheng ZQ, Pilkey WD, Crandall JR, Bass CR, Darvish K 1999. Limiting performance of helmets for the prevention of head injury. Shock and Vibration, 6, 299-320. [2.2]

Chou CC, Nyquist GW 1974. Analytical studies of the Head Injury Criterion (HIC). Technical Paper No. 740082, Society of Automotive Engineers. [4.1.3, 6.5.5, 7.8.1]

Chuma Y, Murata S, Kii T 1970. Measurement of impact acceleration of apple fruits and its analysis by Hertz's impact theory. (In Japanese.) Journal of the Japanese Society of Agricultural Machinery, 32, 47-52. [18.1]

Chuma Y, Shiga T, Iwamoto M 1978. Mechanical properties of satsuma orange as related to the design of a container for bulk transportation. Journal of Texture Studies, 9, 461-479. [18.1]

Clemo KC, Davies RG 1998. The practicalities of pedestrian protection. Paper 98-S10-P-16, 16th International Technical Conference on the Enhanced Safety of Vehicles. [2.2]

Coats B, Margulies SS 2008. Potential for head injuries in infants from low-height falls: Laboratory investigation. Journal of Neurosurgery: Pediatrics, 2(11), 321-330. [14.7]

Craig V 1993. Analysis of pole barrier test data and impact equations. Accident Reconstruction Journal, 5(5), pp. 50, 51, 64. [17.9]

Cripton PA, Dressler DM, Stuart CA, Dennison CR, Richards D 2014. Bicycle helmets are highly effective at preventing head injury during head impact: Head-form accelerations and injury criteria for helmeted and unhelmeted impacts. Accident Analysis and Prevention, 70, 1-7. [14.3.1, 14.4.1]

Daum M 2006. A simplified process for determining cushion curves: The stress-energy method. Presented at the Dimensions.06 Conference. http://www.talkpkg.com/Papers-Presentations/index.htm [17.4.2]

Deb A, Ali T 2004. A lumped parameter-based approach for simulation of automotive headform impact with countermeasures. International Journal of Impact Engineering, 30, 521-539. [4.2.3]

de Bruyn JR, Walsh AM 2004. Penetration of spheres into loose granular media. Canadian Journal of Physics, 82, 439-446. [15.2, 15.9]

Deitz PH 1998. A V/L taxonomy for analyzing ballistic live-fire events. Proceedings of the 15th International Symposium on Military Operational Research. http://ismor.cds.cranfield.ac.uk/15th-symposium-1998 [19.4.5]

Delye H, Verschueren P, Depreitere B, Verpoest I, Berckmans D, Vander Sloten J, Van der Perre G, Goffin J 2007. Biomechanics of frontal skull fracture. Journal of Neurotrauma, 24, 1576-1586. [5.2.3]

DeMarco AL, Chimich DD, Gardiner JC, Nightingale RW, Siegmund GP 2010. The impact response of motorcycle helmets at different impact severities. Accident Analysis and Prevention, 42, 1778-1784. [7.10]

DeWeese RL, Moorcroft DM 2004. Evaluation of a Head Injury Criteria Component Test Device. Report DOT/FAA/AM-18, FAA Civil Aerospace Medical Institute, Federal Aviation Administration, Oklahoma City. [5.3.1, 11.6]

Dickson K, Strunk W, Sorochan J 2018. The effect of soil type and moisture content on head impacts on natural grass athletic fields. Proceedings, 2, paper 270 (12th Conference of the International Sports Engineering Association). [9.4.2]

Digges KH 1998. Injury measurements and criteria. In: Models for Aircrew Safety Assessment: Uses, Limitations and Requirements, http://ftp.rta.nato.int/public/PubFullText/RTO/MP/RTO-MP-020/$MP-020-$K2.PDF [4.2.2]

Dionne J-P, Wong D, Halpin S, Levine J, Makris A 2010. Helmet-mounted blast dosimeter for the military. Electronics and signal processing challenges. Presented at the 3rd International Symposium on Applied Sciences in Biomedical and Communication Technologies, held in Rome, Italy. [12.6.4]

Dutschke JK 2013. Alternative injury response functions for evaluating head accelerations, with application to pedestrian headform testing. Ph.D. thesis, Centre for Automotive Safety Research, University of Adelaide. [4.1.1, 4.2.3]

Eager D, Hayati H, Chapman C 2016. Impulse force as an additional safety criterion for improving the injury prevention performance of impact attenuation surfaces in children's playgrounds. Paper IMECE2016-65565, Proceedings of the International Mechanical Engineering Congress and Exposition, American Society of Mechanical Engineers. [10.8]

Edwards WB, Meardon SA, Ward ED, Derrick TR 2006. The use of acceleration and external forces to estimate bone strain.

Proceedings of the American Society of Biomechanics Conference. http://www.asbweb.org/conferences/2006/pdfs/261.pdf [11.8.1]

Eisler RD, Chatterjee AK, Burghart GH, O'Keefe JA 2001. Casualty assessments of penetrating wounds from ballistic trauma. Technical Report NATICK/TR-01/011, Mission Research Corporation, Costa Mesa, California. [15.7]

Fan WRS 1983. Head impacts with roof-pillar support structures --- Problem identification. Technical Paper No. 831620, Society of Automotive Engineers. [7.9]

Feraboli P, Kedward KT 2004. Enhanced evaluation of the low-velocity impact response of composite plates. AIAA Journal, 42, 2143-2152. [4.6.4]

Flores P, Lankarani HM 2016. Dissipative contact force models. Chapter 3 of Contact Force Models for Multibody Dynamics. Solid Mechanics and Its Applications, 226, 27-52. http://dx.doi.org/10.1007/978-3-319-30897-5_3 [4.2.5]

Forrestal MJ, Frew DJ, Hanchak SJ, Brar NS 1996. Penetration of grout and concrete targets with ogive-nose steel projectiles. International Journal of Impact Engineering, 18, 465-476. [16.5.2]

Forrestal MJ, Frew DJ, Hickerson JP, Rohwer TA 2003. Penetration of concrete targets with deceleration-time measurements. International Journal of Impact Engineering, 28, 479-497. [16.5.2]

Franklyn M, Laing S 2016. Evaluation of military helmets and roof padding on head injury potential from vertical impacts. Traffic Injury Prevention, 17, 750-757. [12.2]

Fu K, Wang H, Chang L, Foley M, Friedrich K, Ye L 2018. Low-velocity impact behaviour of a shear thickening fluid (STF) and STF-filled sandwich composite panels. Composites Science and Technology, 165, 74-83. [17.18]

Fujikawa T, Sugiura R, Nishikata R, Nishimoto T 2017. Critical contact pressure and transferred energy for soft tissue injury by blunt impact in human-robot interaction. Proceedings of the 17th International Conference on Control, Automation and Systems, held in Jeju, Korea, pp. 867-872. [13.7.3]

Fukaya K, Uchida M 2008. Protection against impact with the ground using wearable airbags. Industrial Health, 46, 59-65. [9.2.4, 11.1.1]

Gabler LF, Crandall JR, Panzer MB 2016. Investigating brain injury tolerance in the sagittal plane using a finite element model of the human head. International Journal of Automotive Engineering, paper 20164082. [5.2.3]

Gao D, Irwin AL 2015. Using a Hybrid III anthropomorphic test device with back constraint to evaluate robot impact to the chest. Proceedings of the IEEE International Conference on Robotics and Biomimetics, held in Zhuhai, China, pp. 2466-2471. [13.7.2]

Gao D, Wampler CW 2009. Head Injury Criterion. Assessing the danger of robot impact. IEEE Robotics and Automation Magazine, 16(4), 71-74. [4.2.3]

Genge RA, Bilanski WK, Menzies DR 1978. Test methods and performance of some fruit catching surfaces. Transactions of the ASAE (American Society of Agricultural Engineers), 21, 249-252. [18.1]

Ghajari M, Galvanetto U 2010. Virtual and experimental testing of helmets. Report from Department of Aeronautics, Imperial College London. European Commission DG Research, Sixth Framework Programme, MYMOSA: Motorcycle and Motorcyclist Safety. [7.12]

Ghani ANA, Rased ANWA 2014. Load absorption characteristics of tyre production waste rubber for playground floor systems. Presented at the Advanced Green Material and Technology Symposium, held in Penang. MATEC Web of Conferences, 17, paper 01015. [10.3]

Gilchrist A, Mills NJ 1994. Modelling of the impact response of motorcycle helmets. International Journal of Impact Engineering, 15, 201-218. [8.4.4, 19.1]

Gimbel GM, Hoshizaki TB 2008. Compressive properties of helmet materials subjected to dynamic impact loading of various energies. European Journal of Sport Science, 8, 341-349. [8.4.1, 10.1]

Gockowiak KLK, Anderson RWG, Searson DJ 2014. A model for determining injury risk on the basis of impact speed, using vehicle data from variable-speed impact tests. Proceedings of the IRCOBI Conference, held in Berlin. http://www.ircobi.org/wordpress/downloads/irc14/default.htm [17.9]

Gonthier Y, McPhee J, Lange C, Piedbœuf J-C 2004. A regularized contact model with asymmetric damping and dwell-time dependent friction. Multibody System Dynamics, 11, 209-233. [4.2.5]

Gunatilaka AH, Sherker S, Ozanne-Smith J 2004. Comparative performance of playground surfacing materials including conditions of extreme non-compliance. Injury Prevention, 10, 174-179. [10.4.1]

Haddadin S, Albu-Schäffer A, Frommberger M, Rossmann J, Hirzinger G 2009. The "DLR crash report": Towards a standard crash-testing protocol for robot safety --- Part I: Results. Proceedings of the IEEE International Conference on Robotics and Automation, held at Kobe, Japan, pp. 272-279. [14.6]

Hamacher M, Eckstein L, Kühn M, Hummel T 2011. Assessment of active and passive technical measures for pedestrian protection at the vehicle front. Paper 11-0057, 22nd Enhanced Safety of Vehicles Conference. [19.3.2]

Han I 2007. Non-linear continuous-contact-force model for low-speed front-to-rear vehicle impact. Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering, 221, 1197-1208. [4.2.5, 17.9]

Han I 2016. Simulation analysis of low-speed front-to-rear vehicle impacts for estimating the severity of passenger injury. Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering, 230, 554-563. [17.9]

Han I, Kang H 2016. Determination of the collision speed of a vehicle from evaluation of the crush volume using photographs. Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering, 230, 479-490. [17.9]

Han YH, Lee YW 2003. Development of a vehicle structure with enhanced pedestrian safety. Technical Paper No. 2003-01-1232, Society of Automotive Engineers. [2.2, 7.5]

Harris J 1976. Research and development towards improved protection for pedestrians struck by cars. Proceedings of the 6th International Technical Conference on Experimental Safety Vehicles, pp. 724-734. Washington, D.C.: National Highway Traffic Safety Administration. [2.2]

Herbert RG, McWhannell DC 1977. Shape and frequency composition of pulses from an impact pair. Journal of Engineering for Industry, 99, 513-518. [4.2.5]

Herold B, Truppel I, Siering G, Geyer M 1996. A pressure measuring sphere for monitoring handling of fruit and vegetables. Computers and Electronics in Agriculture, 15, 73-88. [18.5]

Ho TS, Masuya H 2013a. Finite element analysis of the dynamic behavior of sand-filled geocells subjected to impact load by rockfall. International Journal of Erosion Control Engineering, 6, 1-12. [17.11.2]

Ho TS, Masuya H 2013b. An experimental study on the dynamic characteristics of steel beams using sand cushion subjected to falling weight. Journal of Structural Engineering (Kozo Kogaku Ronbunshu), 59A, 1017-1024. [17.11.3]

Hobbs CA, Lawrence GJL, Clarke CS 1985. PSC1 --- A demonstration car with improvements for pedestrian protection. Proceedings of the 10th International Technical Conference on Experimental Safety Vehicles, pp. 31-38. Washington, D.C.: National Highway Traffic Safety Administration. [2.2]

Hopping JA, Zheng JQ, Bosselman S, Masters K, Perritt C 2010. Predictions of helmet pad suspension system performance using isolated pad impact results. Presented at the Personal Armour System Symposium. http://www.dtic/mil/dtic/tr/fulltext/u2/a585967.pdf [12.1]

Horsch JD 1987. Evaluation of occupant protection from responses measured in laboratory tests. Technical Paper No. 870222, Society of Automotive Engineers. [19.2]

Horsch JD, Lau IV, Viano DC, Andrzejak DV 1985. Mechanism of abdominal injury by steering wheel loading. Technical Paper No. 851724, Society of Automotive Engineers. [13.6.1]

Hrysomallis C 2004. Impact energy attenuation of protective football headgear against a yielding surface. Journal of Science and Medicine in Sport, 7, 156-164. [8.3.1]

Hunt KH, Crossley FRE 1975. Coefficient of restitution interpreted as damping in vibroimpact. Journal of Applied Mechanics, 42, 440-445. [4.1.4, 4.2.5, 4.3.1]

Hutchinson TP 2013. Dependence of the Head Injury Criterion and maximum acceleration on headform mass and initial velocity in

tests simulating pedestrian impacts with vehicles. Journal of Biomechanical Engineering, 135, paper 114508. [3.1, 3.8, 4.2.5, 4.3.1, 4.4, 4.5.1, 6.2, 13.2, 13.8]
Hutchinson TP 2014a. Injury mitigation by cushioning impact. Journal of Battlefield Technology, 17(1), 9-13. [11.5.2]
Hutchinson TP 2014b. Letter to the Editor. Military Medicine, 179(4), x. [16.3]
Hutchinson TP 2014c. Peak acceleration during impact with helmet materials: Effect of impactor mass and speed. European Journal of Sport Science, 14, Supplement 1, S377-S382. [8.4.1]
Hutchinson TP 2014d. Equations for depth of penetration of projectiles. (Letter.) Journal of the Royal Army Medical Corps, 160, 261. [16.1.1]
Hutchinson TP 2015a. Accelerations relevant to blunt trauma: Theory and data. Industrial Health, 53, 176-177. [10.9.1]
Hutchinson TP 2015b. Experimental injury: Inference from proxy observations in a test to the real-world average. Journal of Battlefield Technology, 18(1), 1-6. [19.4.5]
Hutchinson TP 2016. A causal model of crash test pulses: Effect of impact speed on deformation and other variables. Australian Journal of Mechanical Engineering, 14, 115-119. [17.9]
Hutchinson TP 2018. Road Safety Theory. http://RoadSafetyTheory.com. [1.1, 1.2, 2.1, 3.1, 4.1.1, 7.1, 19.1.1, 19.3.2, 19.4.4]
Hutchinson TP, Anderson RWG, Searson DJ 2012. Pedestrian headform testing: Inferring performance at impact speeds and for headform masses not tested, and estimating average performance in a range of real-world conditions. Traffic Injury Prevention, 13, 402-411. [19.1.1, 19.3.1]
Hutchinson TP, Anderson RWG, Searson DJ 2013. An equation for generalising from impact test performance to real-world crashes. Paper 13-0355, 23rd Enhanced Safety of Vehicles Conference. [19.3.2, 19.4.4]
Hutchinson TP, Anderson RWG, Searson DJ 2016. Regulatory testing for safety: The mathematics of broad-based results. Presented at the 12th Biennial Engineering Mathematics and Applications Conference EMAC-2015. ANZIAM Journal (Australian and New Zealand Industrial and Applied Mathematics Journal), 57, Electronic Supplement, C193-C207. [19.1.1]
Hutchinson TP, Searson DJ, Anderson RWG, Dutschke JK, Ponte G, van den Berg AL 2011. Protection of the unhelmeted head against blunt impact: The pedestrian and the car bonnet. Proceedings of the Australasian Road Safety Research, Policing and Education Conference, http://arsrpe.acrs.org.au/index.cfm?action=main.paper&id=2300 [2.1, App2]
Isa MT, Ahmed AS, Aderemi BO, Taib RM, Akil HM, Mohammed-Dabo IA 2014. Drop weight impact studies of woven fibers reinforced

modified polyester composites. Leonardo Electronic Journal of Practices and Technologies, No. 24, 97-112. [17.15]

Ito D, Yamada H, Oida K, Mizuno K 2014. Finite element analysis of kinematic behavior of cyclist and performance of cyclist helmet for human head injury in vehicle-to-cyclist collision. Proceedings of the IRCOBI Conference, held in Berlin. http://www.ircobi.org/wordpress/downloads/irc14/default.htm [7.7]

Iwamoto M, Shiga T, Chuma Y 1976. Effects of dropping and waxing practices in the packing-house line on the quality of satsuma mandarin (*Citrus unshiu* Marc.). (In Japanese.) Journal of the Japanese Society for Horticultural Science, 45, 203-209. [18.1]

Jacobs DA, Waldron KJ 2015. Modeling inelastic collisions with the Hunt-Crossley model using the energetic coefficient of restitution. Journal of Computational and Nonlinear Dynamics, 10, paper 021001. [4.2.5]

Jang S-M, Adachi T, Yamaji A, Kook J-S 2002. Comparing damage in CFRP laminates due to soft body and hard body impacts. Materials Science Research International, 8, 151-155. [4.2.4, 17.15]

Jeong DY, Tang YH, Perlman AB 2011. Semi-analytical approach to estimate railroad tank car shell puncture. Paper JRC2011-56028, Proceedings of the ASME/ASCE/IEEE 2011 Joint Rail Conference. [4.2.4, 17.12]

Ji L, Dang X, Lan B, Song Q, Duan D, Ji Z, Zhang Y, Heng L 2014. Same impact momentum causes different degrees of spinal cord injury. Chinese Science Bulletin, 59, 4298-4303. [4.5.2]

Jiang J-H, Wang Z-W 2012. Dropping damage boundary curves for cubic and hyperbolic tangent packaging systems based on key component. Packaging Technology and Science, 25, 397-411. [17.5.2, 17.5.3]

Johnston C, Maw S, Fauvel R 2006. The design and implementation of a crash pad evaluation system for speed skating. In: E. F. Moritz and S. Haake (Editors), The Engineering of Sport 6, 2, pp. 307-312. New York: Springer. [8.1.3]

Jussila J, Leppäniemi A, Paronen M, Kulomäki E 2005. Ballistic skin simulant. Forensic Science International, 150, 63-71. [16.2]

Kanianthra JN, Rains GC, Trella TJ 1993. Strategies for passenger car designs to improve occupant protection in real world side crashes. Technical Paper 930482, Society of Automotive Engineers. [19.2, App1]

Katagiri M, Katagata K, Pramudita JA, Ujihashi S 2012. Development and application of stress-based skull fracture criteria using a head finite element model. Journal of Biomechanical Science and Engineering, 7, 449-462. [5.2.1, 5.3.1]

Kato M, Shimodaira Y, Sato T, Iida H 2014. Simplification and transformation of ASTM F1292 measurement procedure for fall accident injury criteria. Industrial Health, 52, 407-413. [10.9.1]

Kawano S, Iwamoto M, Hayakawa A, Yamane A, Andou K 1984. Mechanical properties of Japanese pear "Nijisseiki" as related to susceptibility of the fruit to mechanical injury. (In Japanese.)

Journal of the Japanese Society of Agricultural Machinery, 46, 627-632. [18.1]
Kaufman MB, Moss LLC 2015. Reassessing the representative heuristic of the plywood ballistic mannequin used in live-fire testing. Report ARL-TR-7274, U.S. Army Research Laboratory, Aberdeen Proving Ground, Maryland. [16.1.5]
Kessler J, Monk M 1989. NHTSA pedestrian head injury mitigation research program --- Status report. Proceedings of the 12th International Technical Conference on Experimental Safety Vehicles, pp. 1226-1237. Washington, D.C.: National Highway Traffic Safety Administration. [7.8.2]
Kim S-H, Ji S-W, Yoon J-S, Choi S-K, Seo C-H 2011. Relationship of test methods of impact absorbing effect of floors from a viewpoint of safety in accidental collisions. (In Korean.) Journal of the Korea Institute of Building Construction, 11, 29-34. [10.10.1, 10.10.3, 10.10.4]
King AI 2004. Injury biomechanics from head to foot. In: P. Prasad and J. E. Belwafa (Editors), Vehicle Crashworthiness and Occupant Protection, pp. 269-352. Southfield, Michigan: American Iron and Steel Institute. [3.8]
Kitazawa H, Saito K, Ishikawa Y 2014. Effect of difference in acceleration and velocity change on product damage due to repetitive shock. Packaging Technology and Science, 27, 221-230. [18.6]
Kleiven S, von Holst H 2002. Consequences of head size following trauma to the human head. Journal of Biomechanics, 35, 153-160. [4.6.1]
Kokinakis W, Sperrazza J 1965. Criteria for incapacitating soldiers with fragments and flechettes. Report 1269, Ballistic Research Laboratories, Aberdeen Proving Ground, Maryland. [19.4.5]
Komarnicki P, Stopa R, Szyjewicz D, Kuta Ł, Klimza T (2017). Influence of contact surface type on the mechanical damages of apples under impact loads. Food and Bioprocess Technology, 10, 1479-1494. [18.1]
Korner J 1989. A method for evaluating occupant protection by correlating accident data with laboratory test data. Technical Paper No. 890747, Society of Automotive Engineers. [19.2]
Krebsbach MA, Muci-Küchler KH, Hinz BJ 2012. Effect of initial projectile speed on contamination distribution in a lower extremity surrogate "wound track". Military Medicine, 177, 573-582. [16.3]
Kuehnel A, Appel H 1978. First step to a pedestrian safety car. Technical Paper 780901, Society of Automotive Engineers. [2.2]
Kurt M, Laksari K, Kuo C, Grant GA, Camarillo DB 2017. Modeling and optimization of airbag helmets for preventing head injuries in bicycling. Annals of Biomedical Engineering, 45, 1148-1160. [14.4.4]
Kuutti J, Hakola I, Fortino S 2012. Analyses of disposal canister falling accidents. Posiva Report 2012-36, Posiva Oy, Eurajoki, Finland. http://www.posiva.fi/en/databank [15.9, 16.5.4]
Lambert JP, Jonas GH 1976. Towards standardization in terminal ballistics testing: Velocity representation. Report No. 1852, USA

Ballistic Research Laboratories, Aberdeen Proving Ground, Maryland. [16.1.5, 16.6]

Lau IV, Horsch JD, Viano DC, Andrzejak DV 1987. Biomechanics of liver injury by steering wheel loading. Journal of Trauma, 27, 225-235. [13.6.1]

Lau IV, Viano DC 1986. The Viscous Criterion --- Bases and applications of an injury severity index for soft tissues. Technical Paper No. 861882, Society of Automotive Engineers. [13.1, 13.3, 13.4]

Lawrence GJL, Hardy BJ, Carroll JA, Donaldson WMS, Visvikis C, Peel DA 2006. A study on the feasibility of measures relating to the protection of pedestrians and other vulnerable road users. TRL Project Report UPR/VE/045/06, TRL Limited, Berkshire, U.K. [2.2, App2]

Li Z, Miao F, Andrews J 2017. Mechanical models of compression and impact on fresh fruits. Comprehensive Reviews in Food Science and Food Safety, 16, 1296-1312. [18.2]

Lin C-Y, Chuang H-J, Cortes DH 2017. Investigation of the optimum heel pad stiffness: A modeling study. Australasian Physical and Engineering Sciences in Medicine, 40, 585-593. [11.8.2]

Li Piani T, Weerheijm J, Koene L, Sluys LJ 2017. The ballistic resistance of adobe masonry: An analytical model for impacts on mud bricks and mortar. Proceedings of the 17th International Symposium on the Interaction of the Effects of Munitions with Structures. [15.7]

Li Piani T, Weerheijm J, Sluys LJ 2018. Ballistic model for the prediction of penetration depth and residual velocity in adobe: A new interpretation of the ballistic resistance of earthen masonry. Defence Technology, to be published. [15.7]

Long J, Yang J, Lei Z, Liang D 2015. Simulation-based assessment for construction helmets. Computer Methods in Biomechanics and Biomedical Engineering, 18, 24-37. [11.4]

Lu L-X, Wang Z-W 2007. Dropping bruise fragility and bruise boundary of apple fruit. Transactions of the ASABE (American Society of Agricultural and Biological Engineers), 50, 1323-1329. [18.4]

Lyn G, Mills NJ 2001. Design of foam crash mats for head impact protection. Sports Engineering, 4, 153-163. [8.2]

MacLaughlin TF, Kessler JW 1990. Pedestrian head impact against the central hood of motor vehicles --- Test procedure and results. Technical Paper No. 902315, Society of Automotive Engineers. [7.5]

Makris A, Nerenberg J, Dionne JP, Bass CR, Chichester C 2000. Reduction of blast induced head acceleration in the field of anti-personnel mine clearance. Proceedings of the 2000 International IRCOBI Conference on the Biomechanics of Impact, pp. 453-464. Bron, France: IRCOBI. [12.6.1, 12.6.2, 12.6.3, 12.6.4]

Malasri S, Moats R, Archer J, Shiue P, Brown R, Rutledge L 2012. Plastic tote drop impact study. Journal of Packaging, Institute of Packaging Professionals, January 2012. https://www.iopp.org/files/public/PlasticToteDropImpactStudy2.pdf [17.7]

Marshall DE, Brook RC 1999. Reducing bell pepper bruising during postharvest handling. HortTechnology, 9, 254-258. [18.1]

Martin DA, Obstalecki M, Kurath P, Horn GP 2014. An approach for quantifying dynamic properties and simulated deployment loading of fire service escape rope systems. Experimental Techniques, http://dx.doi.org/10.1111/ext.12073 [11.3]

Martin RB 1990. Problems associated with testing the impact absorption properties of artificial playing surfaces. In: R. C. Schmidt, E. F. Hoerner, E. M. Milner, and C. A. Morehouse (Editors), Natural and Artificial Playing Fields: Characteristics and Safety Features, pp. 77-84. Philadelphia, PA: American Society for Testing and Materials. [4.2.4]

Martin RB, Liptai L, Yerby S, Williams KR 1994. The relationship between mass and acceleration for impacts on padded surfaces. Journal of Biomechanics, 27, 361-364. [3.8, 4.2.4, 9.5]

Martínez AB, Xiang G, Arencón D 2012. Flexed-plate low energy impact of an epoxy-carbon fibre laminate. Polymers and Polymer Composites, 20, 581-587. [17.15]

Masoumi A, Shojaeefard MH, Najibi A 2011. Comparison of steel, aluminum, and composite bonnet in terms of pedestrian head impact. Safety Science, 49, 1371-1380. [7.5]

Masuzawa H, Hirahawa K, Nakamura N, Sano K, Kihira M, Hayashi T 1971. Engineering aspects of human skull fracture. Experimental study and theoretical consideration on brain damage. Neurologia Medico-Chirurgica, 11, 46-65. [14.1]

Mattar Neto M, Miranda CAdeJ, Fainer G 2008. Nonlinear issues on the numerical simulations of the 9 meter drop test of a transportation package. Proceedings of the 29th Iberian Latin American Congress on Computational Methods in Engineering. [17.7]

Maw S, Lun V, Clarke A 2012. The influence of helmet size and shape on peak linear decelerations when impacting crash pads. Proceedings of the 9th Conference of the International Sports Engineering Association: Procedia Engineering, 34, 819-824. [8.1.1, 8.1.2, 8.1.3]

McIntosh AS, Janda D 2003. Evaluation of cricket helmet performance and comparison with baseball and ice hockey helmets. British Journal of Sports Medicine, 37, 325-330. [3.5]

McLean AJ, Brewer ND, Sandow BL 1979. Adelaide in-depth accident study 1975-1979. Part 2: Pedestrian accidents. Report from the Road Accident Research Unit, University of Adelaide. http://casr.adelaide.edu.au/casrpub/show.php?id=14 [2.2]

Mertz HJ 1985. Biofidelity of the Hybrid III head. Technical Paper No. 851245, Society of Automotive Engineers. [14.2, 14.3.1]

Mertz HJ 1993. Anthropomorphic test devices. In: A. M. Nahum and J. W. Melvin (Editors), Accidental Injury. Biomechanics and Prevention, pp. 66-84. New York: Springer. [4.6.1]

Meyer CS, Haque BZ, O'Brien DJ, Getinet N, Yu JH, Bonyi E, Aslan K, Gillespie JW 2018. Mesoscale ballistic damage mechanisms of a

single-layer woven glass/epoxy composite. International Journal of Impact Engineering, 113, 118-131. [16.2, 16.6]
Miao H, Fu J, Qian Z, Ren L, Ren L 2017. How does the canine paw pad attenuate ground impacts? A multi-layer cushion system. Biology Open, 6, 1889-1896. [14.8]
Miller PM, Rychlewski HA, Lee JC 1996. Free motion headform testing: Results and potential design countermeasures. Proceedings of the 15th International Technical Conference on Enhanced Safety of Vehicles, pp. 1291-1301. Washington, DC: NHTSA. [7.6, 7.9]
Mindlin RD 1945. Dynamics of package cushioning. Bell System Technical Journal, 24, 353-461. [4.2.4, 4.3.5, 17.1]
Mizuno K, Ito D, Oida K, Kobayashi G, Han Y 2017. Head protection with cyclist helmet in impact against vehicle A-pillar. International Journal of Crashworthiness, 22, 322-331. [7.3, 7.7]
Mizuno K, Kajzer J 2000. Head injuries in vehicle-pedestrian impact. Technical Paper No. 2000-01-0157, Society of Automotive Engineers. [4.5.1, 7.3, 7.5]
Mizuno K, Yonezawa H, Kajzer J 2001. Pedestrian headform impact tests for various vehicle locations. Proceedings of the 17th Enhanced Safety of Vehicles Conference, paper 0278. http://www-nrd.nhtsa.dot.gov/pdf/esv/esv17/proceed/00185.pdf. This is also Technical Paper No. 2001-06-0185, Society of Automotive Engineers. [7.3, 7.5]
Mizutani H, Kawakami S-i, Takagi Y, Kato M, Kumazawa M 1983. Cratering experiments in sands and a trial for general scaling law. Journal of Geophysical Research: Solid Earth, 88, A835-A845. [16.5.3]
Monea AG, Van der Perre G, Baeck K, Delye H, Verschueren P, Forausebergher E, Van Lierde C, Verpoest I, Vander Sloten J, Goffin J, Depreitere B 2014. The relation between mechanical impact parameters and most frequent bicycle related head injuries. Journal of the Mechanical Behavior of Biomedical Materials, 33, 3-15. [3.3.3, 5.2.3]
Monk MW, Sullivan LK 1986. Energy absorption material selection methodology for head/A-pillar. Technical Paper No. 861887, Society of Automotive Engineers. [7.8.1, 7.8.2, 7.8.4]
Mordaka J, Kleiven S, van Schijndel-de Nooij M, de Lange R, Guerra Casanova LJ, Carter EL, von Holst H 2007. The importance of rotational kinematics in pedestrian head to windshield impacts. Proceedings of the 2007 International IRCOBI Conference on the Biomechanics of Impact, pp. 83-94. http://ircobi.org/wordpress/proceedings/ [5.2.3]
Morye SS, Hine PJ, Duckett RA, Carr DJ, Ward AM 2000. Modelling of the energy absorption by polymer composites upon ballistic impact. Composites Science and Technology, 60, 2631-2642. [5.4.3, 15.9]
Moss WC, King MJ, Blackman EG 2014. Towards reducing impact-induced brain injury: Lessons from a computational study of army

and football helmet pads. Computer Methods in Biomechanics and Biomedical Engineering, 17, 1173-1184. [12.1]

Murayama H, Hitosugi M, Motozawa Y, Ogino M, Koyama K 2013. Simple strategy to prevent severe head trauma in judo --- Biomechanical analysis. Neurologia Medico-Chirurgica, 53, 580-584. [9.2.1, 9.2.3, 9.2.4]

Naick PA, Carnago KM 1998. Theoretical relationship of HIC d to crush space for FMVSS-201 head impact tests, and the effective use of FEA in predicting HIC for design, verification and optimization of countermeasures. Technical Paper No. 982349, Society of Automotive Engineers. [7.9]

Nakanishi Y, Nakamura N, Hasegawa N, Inamori H, Ogawa Y, Kitazawa H 2015. Evaluation and estimation of damage to tree-ripened Irwin mangos from repetitive shock during transportation. Tropical Agriculture and Development, 59, 112-117. [18.6]

Nakano M, Matsuura H, Tamagawa M, Yamanaka M, Yukimasa T 2010. Theoretical analysis of Head Injury Criterion. Proceedings of the Annual Conference of Biomedical Fuzzy Systems Association, pp. 335-340. http://www.jstage.jst.go.jp/article/pacbfsa/23/0/23_335/_article [4.6.2]

Nakano M, Matsuura H, Tamagawa M, Yukimasa T, Yamanaka M 2011. Theoretical analysis of Collision Impact Index --- CII. Proceedings of the Annual Conference of Biomedical Fuzzy Systems Association, pp. 177-180. http://www.jstage.jst.go.jp/article/pacbfsa/24/0/24_177/_article [4.6.2]

Navarro-Javierre P, Garcia-Romeu-Martinez M-A, Cloquell-Ballester V-A, de-la-Cruz-Navarro E 2012. Evaluation of two simplified methods for determining cushion curves of closed cell foams. Packaging Technology and Science, 25, 217-231. [17.4.3]

Neilson ID 1969. Simple representations of car and unrestrained occupant impacts in road accidents. RRL Report LR 249, Road Research Laboratory, Crowthorne, U.K. [4.2.4]

Neilson ID, Kemp RN, Wall JG, Harris J 1968. Controlled impact investigations. Head-on impacts of four similar cars from different speeds against a rigid barrier. RRL Report LR 132, Road Research Laboratory, Crowthorne, U.K. [17.9]

Newman JA 1987. Discussion of SAE Paper No. 870222 by Horsch. Proceedings of the SAE International Congress and Exposition (Restraint Technologies --- Front Seat Occupant Protection), pp. 41-42. Society of Automotive Engineers. [3.8, 13.1]

NHTSA 1993. Pedestrian injury reduction research. Report to the Congress, June 1993. Report DOT HS 808 026. Washington, DC: National Highway Traffic Safety Administration. [4.5.1, 7.5]

NHTSA 1995. Final economic assessment, FMVSS No. 201, upper interior head protection. Washington, DC: National Highway Traffic Safety Administration. [19.3.1]

Nishida M, Nakagawa J, Kuzuya K, Aikawa F, Hayashi K, Tanaka K 2009. Thin plate bumper influence on crater formation in thick aluminum targets. Journal of Space Engineering, 2, 51-60. [16.5.3]

Nsiampa N, Robbe C, Oukara A, Papy A 2012. Comparison of less lethal 40 mm sponge projectile and the 37 mm projectile for injury assessment on human thorax. EPJ Web of Conferences, 26, paper 03002. http://dx.doi.org/10.1051/epjconf/20122603002 [3.3.3, 12.5.3, 12.5.4]

Ohue Y, Miyoshi K 2014. Assessment of Head Injury Criterion score of shock attenuation material employed around playground equipment using nonlinear spring contact model. Open Journal of Safety Science and Technology, 4, 73-83. [4.2.4, 10.7]

Oikawa S, Matsui Y, Wakabayashi A, Gomei S, Nakadate H, Aomura S 2016. Severity of cyclist head injuries caused by impacts with vehicle structure and road surface. Journal of Biomechanical Science and Engineering, 11, paper 15-00613. [7.7]

Oikawa S, Nakadate H, Zhang Y, Ueno T, Aomura S, Matsui Y 2017. Finite element analysis of the effectiveness of bicycle helmets in head impacts against roads. Journal of Biomechanical Science and Engineering, 12, paper 17-00175. [3.3.3, 14.4.3]

Okamoto Y, Akiyama A, Nagatomi K, Tsuruga T 1994. Concept of hood design for possible reduction in pedestrian head injury. Proceedings of the 14th International Technical Conference on Enhanced Safety of Vehicles, Volume 2, pp. 1253-1263. Washington, D.C.: National Highway Traffic Safety Administration. [2.2]

Ommaya AK, Thibault L, Bandak FA 1994. Mechanisms of impact head injury. International Journal of Impact Engineering, 15, 535-560. [3.3.3]

Ouckama R, Pearsall DJ 2012. Impact performance of ice hockey helmets: Head acceleration versus focal force dispersion. Proceedings of the Institution of Mechanical Engineers, Part P: Journal of Sports Engineering and Technology, 226, 185-192. [3.6, 8.6.2]

Ouckama R, Pearsall DJ 2014. Projectile impact testing of ice hockey helmets: Headform kinematics and dynamic measurement of localized pressure distribution. Proceedings of the IRCOBI Conference, held in Berlin. http://www.ircobi.org/wordpress/downloads/irc14/default.htm [3.6, 8.6.1]

Oukara A, Nsiampa N, Robbe C, Papy A 2014. Injury risk assessment of non-lethal projectile head impacts. The Open Biomedical Engineering Journal, 8, 75-83. [3.5]

Oukara A, Robbe C, Nsiampa N, Papy A 2013. Comparison of the lethality of different kinetic energy non-lethal projectiles using a new French assessment approach for head impacts. Proceedings of the 27th International Symposium on Ballistics, pp. 564-575. Lancaster, PA: DEStech. [3.5, 12.3]

Pain MTG, Mills CL, Yeadon MR 2005. Video analysis of the deformation and effective mass of gymnastics landing mats. Medicine and Science in Sports and Exercise, 37, 1754-1760. [9.6]

Park J-J, Song J-B, Haddadin S 2015. Collision analysis and safety evaluation using a collision model for the frontal robot-human impact. Robotica, 33, 1536-1550. [13.7.1]

Park JM, Kim GS, Kwon SH, Chung SW, Kwon SG, Choi WS, Kim JS 2016. Modeling and analysis of cushioning performance for multi-layered corrugated structures. Journal of Biosystems Engineering, 41, 221-231. [17.3.2, 17.4.4]

Pavier J, Langlet A, Eches N, Jacquet J-F 2015a. On ballistic parameters of less lethal projectiles influencing the severity of thoracic blunt impacts. Computer Methods in Biomechanics and Biomedical Engineering, 18, 192-200. [12.4]

Pavier J, Langlet A, Eches N, Prat N, Bailly P, Jacquet J-F 2015b. Experimental study of the coupling parameters influencing the terminal effects of thoracic blunt ballistic impacts. Forensic Science International, 252, 39-51. [12.4]

Peache R 1986. Cushioning, Design. In: The Wiley Encyclopedia of Packaging Technology. New York: Wiley. [19.4.6]

Pearce CW, Young PG 2014. On the pressure response in the brain due to short duration blunt impacts. PLoS ONE, 9, paper e114292. [3.5, 8.7, 14.9]

Peck WC, Beard AP 1931. Drop and flight tests on NY-2 landing gears including measurements of vertical velocities at landing. Report No. 406, National Advisory Committee for Aeronautics, Langley Memorial Aeronautical Laboratory, Langley Field, VA. Available at http://ntrs.nasa.gov [17.10]

Peleg K 1985. Biomechanics of fruits and vegetables. Journal of Biomechanics, 18, 843-862. [18.2]

Pereira NQ 2010. ISO headform pedestrian protection test results comparison at critical bonnet regions. Technical Paper No. 2010-36-0236, Society of Automotive Engineers. [7.3]

Perera S, Lam N, Pathirana M, Zhang L, Ruan D, Gad E (2016). Deterministic solutions for contact force generated by impact of windborne debris. International Journal of Impact Engineering, 91, 126-141. [4.2.4, 17.8]

Perera S, Lam N, Pathirana M, Zhang L, Ruan D, Gad E (2018). Probabilistic modelling of forces of hail. Natural Hazards, 91, 133-153. [17.8]

Petrone N 2012. The effect of impact speed, construction, and layout of different ski safety barriers on peak decelerations and penetration values of a solid dummy during full scale impacts. In: R. J. Johnson, J. E. Shealy, R. M. Greenwald, and I. S. Scher (Editors), Skiing Trauma and Safety: 19th Volume, pp. 153-170. Publication STP 1553. West Conshohocken, PA: ASTM International. [9.2.4]

Piao M, Saito K, Kawaguchi K, Zhong C 2017. Effectiveness verification of the hybrid free-fall test. Packaging Technology and Science, 2017, 1-11. [4.2.5]

Ponte G, van den Berg A, Anderson RWG, Linke B 2013. The pedestrian protection in vehicle impacts: Demystifying pedestrian testing procedures and assessment. Presented at the Australasian College of Road Safety Conference "A Safe System: The Road Safety Discussion", held in Adelaide. http://papers.acrs.org.au/index.cfm?action=main.paper&id=442 [2.2]

Post A, Hoshizaki TB 2012. Mechanisms of brain impact injuries and their prediction: A review. Trauma, 14, 327-349. [5.2.1]

Praeger U, Surdilovic J, Truppel I, Herold B, Geyer M 2013. Comparison of electronic fruits for impact detection on a laboratory scale. Sensors, 13, 7140-7155. [18.5]

Prange MT, Luck JF, Dibb A, Van Ee CA, Nightingale RW, Myers BS 2004. Mechanical properties and anthropometry of the human infant head. Stapp Car Crash Journal, 48, 279-299. [14.2]

Prat N, Rongieras F, de Freminville H, Magnan P, Debord E, Fusai T, Destombe C, Sarron J-C, Voiglio EJ 2012. Comparison of thoracic wall behavior in large animals and human cadavers submitted to an identical ballistic blunt thoracic trauma. Forensic Science International, 222, 179-185. [12.4]

Quenneville CE, Dunning CE 2012. Evaluation of the biofidelity of the HIII and MIL-Lx lower leg surrogates under axial impact loading. Traffic Injury Prevention, 13, 81-85. [11.8.3]

Radomirovic D, Kovacic I 2015. An equivalent spring for nonlinear springs in series. European Journal of Physics, 36, paper 055004. [5.2.2]

Rahman IA, Ahmad Zaidi AM, Bux Q 2010. Review on empirical studies of local impact effects of hard missile on concrete structures. International Journal of Sustainable Construction Engineering and Technology, 1(1), 73-97. [16.5.2]

Rider RC, Fridley RB, O'Brien M 1973. Elastic behavior of a pseudo-fruit for determining bruise damage to fruit during mechanized handling. Transactions of the ASAE (American Society of Agricultural Engineers), 16, 241-244. [18.1]

Rigby P, Juhas B, Wong J, Chan P 2011. Evaluation of biofidelity of ECE Regulation No. 22 injury criteria. Paper 11-0366, 22nd Enhanced Safety of Vehicles Conference. [3.8, 5.2.1]

Ruan JS, Prasad P 2006. On the consequences of head size following impact to the human head. Journal of Biomechanics, 39, 383-385. [4.6.1]

Ruiz-Herrero JL, Rodrìguez-Pérez MA, de Saja JA 2005. Prediction of cushion curves for closed cell polyethylene-based foams. Part I. Modelling. Cellular Polymers, 24, 329-346. [17.4.2]

Ruiz-Herrero JL, Rodrìguez-Pérez MA, de Saja JA 2006. Prediction of cushion curves for closed cell polyethylene-based foams. Part II: Experimental. Cellular Polymers, 25, 159-176. [17.4.2]

Ryu D-M, Wang L, Kim S-K, Lee J-M 2017. Comparative study on deformation and mechanical behavior of corroded pipe: Part I --- Numerical simulation and experimental investigation under impact load. International Journal of Naval Architecture and Ocean Engineering, 9, 509-524. [17.12]

Sanchez-Molina D, Velazquez-Ameijide J, Arregui-Dalmases C, Crandall JR, Untaroiu CD 2012. Minimization of analytical injury metrics for head impact injuries. Traffic Injury Prevention, 13, 278-285. [4.6.1]

Saunders L, Wang J, Slater D, Van den Berg J 2012. Visco-elastic polyurethane foam as an injury mitigation device in military aircraft seating. Journal of Battlefield Technology, 15, 11-18. [11.5.1, 11.5.3]

Savvateev AF, Budin AV, Kolikov VA, Rutberg PhG 2001. High-speed penetration into sand. International Journal of Impact Engineering, 26, 675-681. [15.5, 15.9]

Searle JA, Bethell J, Baggaley G 1978. The variation of human tolerance to impact and its effect on the design and testing of automotive impact performance. Technical Paper No. 780885, Society of Automotive Engineers. [19.2]

Searson DJ, Anderson RWG, Hutchinson TP 2012a. The effect of impact speed on the HIC obtained in pedestrian headform tests. International Journal of Crashworthiness, 17, 562-570. [7.2]

Searson DJ, Anderson RWG, Hutchinson TP 2012b. Use of a damped Hertz contact model to represent head impact safety tests. In: A. Kolousov et al. (Editors), Proceedings of the 7th Australasian Congress on Applied Mechanics, pp. 230-239. Barton, ACT: National Committee on Applied Mechanics, Engineers Australia. (ISBN of USB: 9781922107619.) [4.2.5]

Searson DJ, Anderson RWG, Ponte G, van den Berg AL 2009. Headform impact test performance of vehicles under the GTR on pedestrian safety. Report 072, Centre for Automotive Safety Research, University of Adelaide. http://casr.adelaide.edu.au/casrpub/show.php?id=1139 [4.2.3]

Segletes SB 2008. Modeling the penetration behavior of rigid spheres into ballistic gelatin. Report ARL-TR-4393, U.S. Army Research Laboratory, Aberdeen Proving Ground, Maryland. [15.1, 15.4, 15.9, 16.1.1]

Seidi M, Hajiaghamemar M, Caccese V 2015. Evaluation of effective mass during head impact due to standing falls. International Journal of Crashworthiness, 20, 134-141. [14.5.1, 14.5.2]

Sheehan RL 1988. Predicting field performance from laboratory testing. In: R. J. Peache and D. O'Sullivan (Editors), Current Trends in Protective Packaging of Computers and Electronic Components, ASTM STP (Special Technical Publication) No. 994, pp. 43-57. Philadelphia: American Society for Testing and Materials. [19.4.6]

Shields BJ, Smith GA 2009. The potential for brain injury on selected surfaces used by cheerleaders. Journal of Athletic Training, 44, 595-602. [9.3.1, 9.3.2, 9.3.3]

Shinoda Y, Futase K, Umezaki E 2008. Impact pressure in new containers for liquid products subjected to impact due to dropping. (In Japanese.) Journal of the Japanese Society for Experimental Mechanics, 8, 239-244. [17.17]

Shivakumar KN, Elber W, Illg W 1983. Prediction of impact force and duration during low velocity impact on circular composite laminates. Technical Memorandum 85703, NASA Langley Research Center, Hampton VA. Also published as: Prediction of impact force and duration due to low-velocity impact on circular composite laminates. Journal of Applied Mechanics, 52, 674-680 (1985). [5.5.1]

Shojaeefard MH, Najibi A, Ahmadabadi MR 2014. Pedestrian safety investigation of the new inner structure of the hood to mitigate the impact injury of the head. Thin-Walled Structures, 77, 77-85. [7.5]

Shuaeib FM, Hamouda AMS, Hamdan MM, Radin Umar RS, Hashmi MSJ 2002. Motorcycle helmet. Part II. Materials and design issues. Journal of Materials Processing Technology, 123, 422-431. [8.4.4]

Simms C, Wood D 2009. Pedestrian and Cyclist Impact. A Biomechanical Perspective. Dordrecht: Springer. [2.1]

Sonoda Y, Mokhatar SN, Tokumaru S 2012. Elastic plastic impact response of beam element subjected to low velocity impact load using SPH method. Journal of Japan Society of Civil Engineers, Series A2 (Applied Mechanics), 68, I_373-I_381. [17.13]

Soper WG, Dove RC 1962. Similitude in package cushioning. Journal of Applied Mechanics, 29, 263-266. Discussion, 30, 152-153 (1963). [17.3.2, 17.4.4, 17.4.5]

Sounik DF, Gansen P, Clemons JL, Liddle JW 1997. Head-impact testing of polyurethane energy-absorbing (EA) foams. Technical Paper No. 970160, Society of Automotive Engineers. [6.7.2]

Spronk SWF, Kersemans M, De Baerdemaeker JCA, Gilabert FA, Sevenois RDB, Garoz D, Kassapoglou C, Van Paepegem W 2018. Comparing damage from low-velocity impact and quasi-static indentation in automotive carbon/epoxy and glass/polyamide-6 laminates. Polymer Testing, 65, 231-241. [17.15]

Srivastava AK, Rehkugler GE 1976. Strain rate effects in similitude modelling of plastic deformation of structures subject to impact loading. Transactions of the ASAE (American Society of Agricultural Engineers), 19, 617-621. [17.13]

Stalnaker RL, Mendis KK, Rojanavanich V 1992. The Unified Head Injury Theory. In: A. E. Engin, A. Ertas, and U. Karakas (Editors), Proceedings of the 1992 Engineering Systems Design and Analysis Conference. Volume 2, Bioengineering, pp. 105-113. New York: American Society of Mechanical Engineers. [14.4.1]

Stalnaker RL, Rojanavanich V 1990. A practical application of the translational energy criteria: Evaluation of baseball and softball head impact injury potentials. Proceedings of the International IRCOBI Conference on the Biomechanics of Impacts, pp. 225-243. Bron, France: IRCOBI (International Research Council on the Biomechanics of Injury). [8.5]

Stone B, Halkon B, Harland A 2016. Headform mounting performance in cricket standard testing. Proceedings of the 11th Conference of the International Sports Engineering Association: Procedia Engineering, 147, 401-406. [8.5]

Sturdivan LM 1978. A mathematical model of penetration of chunky projectiles in a gelatin tissue simulant. Technical Report ARCSL-TR-78055, Chemical Systems Laboratory, Aberdeen Proving Ground, Maryland. [15.3, 15.6, 15.9, 16.1.1, 16.1.5]

Suhir E 1997. Is the maximum acceleration an adequate criterion of the dynamic strength of a structural element in an electronic product? IEEE Transactions on Components, Packaging, and Manufacturing Technology, Part A, 20, 513-517. [3.1, 3.8, 17.1]

Sun J, Lam N, Zhang L, Ruan D, Gad E 2015. Contact forces generated by hailstone impact. International Journal of Impact Engineering, 84, 145-158. [4.2.4, 17.8]

Suzuki K, Namba K, Watanabe Y 2016. Study on dynamics of penetrator into ice. Transactions of the JSASS (Japan Society for Aeronautical and Space Sciences), Aerospace Technology Japan, 14, Pk_65-Pk_71. [16.5.3]

Takeda F, Yang WQ, Li C, Freivalds A, Sung K, Xu R, Hu B, Williamson J, Sargent S 2017. Applying new technologies to transform blueberry harvesting. Agronomy, 7, paper 33. [18.8]

Tan XG, Saunders RN, Matic P 2017. Combat helmet pad suspension performance for anthropomorphic fit designs, brain functional areas and injury considerations. Paper IMECE2017-70619, Proceedings of the ASME 2017 International Mechanical Engineering Congress and Exposition. [19.4.5]

Tang YH, Yu H, Gordon JE, Jeong DY, Perlman AB 2008. Analysis of railroad tank car shell impacts using finite element method. Paper JRC2008-63014, Proceedings of the 2008 IEEE/ASME Joint Rail Conference. [4.2.4]

Theobald P, Whitelegg L, Nokes LDM, Jones MD 2010. The predicted risk of head injury from fall-related impacts on to third-generation artificial turf and grass soccer surfaces: A comparative biomechanical analysis. Sports Biomechanics, 9, 29-37. [9.4.1, 9.4.2, 9.4.3]

Thota N, Epaarachchi J, Lau KT 2015. Evaluation of the blunt thoracic trauma caused by solid sports ball impacts. Journal of Biomechanical Science and Engineering, 10, paper 14-00264. [13.8]

Tierney G, Thomson R 2003. Methods for assessing the cushioning performance of free-stall dairy cow synthetic beds. Transactions of the ASAE (American Society of Agricultural Engineers), 46, 147-153. [11.8.4]

Timoshenko SP, Goodier JN 1970. Theory of Elasticity. 3rd edition. New York: McGraw-Hill. [4.2.4, 4.3.2]

Trella TJ, Gabler HC, Kanianthra JN, Wagner JJ 1991. Side impact crashworthiness design: Evaluation of padding characteristics through mathematical simulations. Technical Paper No. 912900, Society of Automotive Engineers. [App1]

Tse KM, Tan LB, Yang B, Tan VBC, Lee HP 2017. Effect of helmet liner systems and impact directions on severity of head injuries sustained in ballistic impacts: A finite element (FE) study. Medical and Biological Engineering and Computing, 55, 641-662. [3.3.3]

Unuigbe OM, Onuoha SN 2013. Assessment of impact damage to apple fruits. Nigerian Journal of Technology, 32, 137-140. [18.13]

VanAmburg R 2011. An approach to analyze personnel injury of reflective spall from small-arms protective body armor. Report ARL-TR-5595, U.S. Army Research Laboratory, Aberdeen Proving Ground, Maryland. [19.4.5]

Vander Vorst M, Chan P, Zhang J, Yoganandan N, Pintar F 2004. A new biomechanically-based criterion for lateral skull fracture. Annual Proceedings of the Association for the Advancement of Automotive Medicine, 48, 181-195. [5.3.1, 11.6]

Vander Vorst M, Stuhmiller J, Ho K, Yoganandan N, Pintar F 2003. Statistically and biomechanically based criterion for impact-induced skull fracture. Annual Proceedings of the Association for the Advancement of Automotive Medicine, 47, 363-381. [5.3.1, 5.3.2, 5.3.3, 11.6]

van Mourik S, Alders BPGJ, Helderman F, van de Ven LJF, Groot Koerkamp PWG 2017. Predicting hairline fractures in eggs of mature hens. Poultry Science, 96, 1956-1962. [18.13]

Velmurugan R, Muralikannan R 2009. Energy absorption characteristics of annealed steel tubes of various cross sections in static and dynamic loading. Latin American Journal of Solids and Structures, 6, 385-412. [17.14]

Viano DC 1988. Limits and challenges of crash protection. Accident Analysis and Prevention, 20, 421-429. [19.2]

Viano DC, Withnall C, Wonnacott M 2012. Football helmet drop tests on different fields using an instrumented Hybrid III head. Annals of Biomedical Engineering, 40, 97-105. [9.1.1, 9.1.5]

Vidair C, Haas R, Schlag R 2007a. Evaluating the risk of serious head injury due to falls on California playground surfaces made of recycled tires. Chapter 11 of Evaluation of Health Effects of Recycled Waste Tires in Playground and Track Products. Sacramento, CA: Integrated Waste Management Board. [10.2.1, 10.2.3, 10.2.4]

Vidair C, Haas R, Schlag, R 2007b. Testing impact attenuation on California playground surfaces made of recycled tires. International Journal of Injury Control and Safety Promotion, 14, 225-230. [10.2.1, 10.2.3, 10.2.4]

Wakeland HH 1962. Systematic automobile design for pedestrian injury prevention. In: M. K. Cragun (Editor), (Proceedings of the 5th) Stapp Automotive Crash and Field Demonstration Conference, pp. 193-218. Minneapolis: University of Minnesota. [2.2]

Wall JG, Kemp RN, Harris J 1970. Comparative head-on impact tests of cars with either front transverse or rear mounted engines. RRL Report LR 155, Road Research Laboratory, Crowthorne, U.K. [17.9]
Wang JT 1989. Analytical studies of injury criteria for the thorax. Journal of Biomechanical Engineering, 111, 128-135. [13.1, 13.5]
Wang Z-W 2002a. On evaluation of product dropping damage. Packaging Technology and Science, 15, 115-120. [17.5.1]
Wang Z-W 2002b. Dropping damage boundary curves for cubic and tangent package cushioning systems. Packaging Technology and Science, 15, 263-266. [17.5.1]
Wang Z-W, Jiang J-H 2010. Evaluation of product dropping damage based on key component. Packaging Technology and Science, 23, 227-238. [17.5.2]
Warren LR, Harley S, Dutschke J, van den Berg A, Dobbins C 2017. Bicycle helmet use to reduce the impact of head injuries in ladder users. ANZ Journal of Surgery, 87, 1026-1029. [14.4.2]
Watanabe K, Umezaki E, Futase K 2009. Deformation behavior of liquid-packaging bags under drop impact. Journal of the Japanese Society for Experimental Mechanics, 9, Special Issue s95-s99. [17.17]
Whiteside DJ 2010. A comprehensive overview of the pedestrian protection requirements and test methodology. Technical Paper No. 2010-01-1012, Society of Automotive Engineers. [2.2]
Wierzbicki T, Suh MS 1988. Indentation of tubes under combined loading. International Journal of Mechanical Sciences, 30, 229-248. [4.2.4, 17.12]
Wollert W, Blödorn J, Appel H, Kühnel A 1983. Realization of pedestrian protection measures on cars. Technical Paper No. 830051, Society of Automotive Engineers. [2.2]
Wood DP, Ydenius A, Adamson D 2003. Velocity changes, mean accelerations and displacements of some car types in frontal collisions. International Journal of Crashworthiness, 6, 591-603. [17.9]
Woolam WE 1968. A study of the dynamics of low energy cushioning materials using scale models. Journal of Cellular Plastics, 4, 79-83. [17.3.2]
Woolley RL 2001. Non-linear damage analysis in accident reconstruction. Technical Paper No. 2001-01-0504, Society of Automotive Engineers. [17.9]
Wright AD, Laing AC 2012. The influence of headform orientation and flooring systems on impact dynamics during simulated fall-related head impacts. Medical Engineering and Physics, 34, 1071-1078. [11.2.1]
Wyskida RM, McDaniel DM 1980. Modeling of Cushioning Systems. New York: Gordon and Breach. [17.3.1]
Xie Z, Zhao W, Wang X, Hang J, Yue X, Zhou X 2018. Low-velocity impact behaviour of titanium honeycomb sandwich structures. Journal of Sandwich Structures and Materials, to be published. [17.16]

Xu R, Li C 2015. Development of the second generation Berry Impact Recording Device (BIRD II). Sensors, 15, 3688-3705. [18.8]

Xu R, Takeda F, Krewer G, Li C 2015. Measure of mechanical impacts in commercial blueberry packing lines and potential damage to blueberry fruit. Postharvest Biology and Technology, 110, 103-113. [18.7, 18.8]

Yoshida S, Igarashi N, Takahashi A, Imaizumi I 1999. Development of a vehicle structure with protective features for pedestrians. Technical Paper No. 1999-01-0075, Society of Automotive Engineers. [2.2]

Yu P, Li C, Takeda F, Krewer G 2014. Visual bruise assessment and analysis of mechanical impact measurement in southern highbush blueberries. Applied Engineering in Agriculture, 30, 29-37. [18.7]

Yuan JK, Li YR, Huang RQ, Pei XJ 2015. Impact of rockfalls on protection measures: An experimental approach. Natural Hazards and Earth System Sciences, 15, 885-893. [17.11.1]

Zander O, Gehring D-U, van Ratingen M 2015. Beyond safety legislation: Contribution of consumer information programmes to enhanced injury mitigation of pedestrians during accidents with motor vehicles. Paper 15-0258, 24th Enhanced Safety of Vehicles Conference. [2.2]

Zellmer H 1993. Investigation of the performance of motorcycle helmets under impact conditions. Technical Paper No. 933113, Society of Automotive Engineers. [6.5.5]

Zhang J, Pintar FA, Yoganandan N, Gennarelli TA, Son SF 2009. Experimental study of blast-induced traumatic brain injury using a physical head model. Stapp Car Crash Journal, 53, 215-227. [5.2.1]

Zhang J, Wang F, Yang S, Zhang Y, Song H, Li M 2015. Analysis of drop impact behaviour of eggs with different drop height and cushion material. In: Y. Wu and W. Deng (Editors), Proceedings of the AASRI International Conference on Industrial Electronics and Applications, pp. 36-39. Amsterdam: Atlantis Press. [18.11]

Zhang J, Yoganandan N, Pintar FA 2009. Dynamic biomechanics of the human head in lateral impacts. Annals of Advances in Automotive Medicine, 53, 249-256. [11.7]

Zhang S, Ocakli H, Pedersen PT 2004. Crushing of ship bows in head-on collision. International Journal of Maritime Engineering, 146, 39-46. [4.2.4]

Zhou CY, Yu TX, Lee RSW 2008. Drop/impact tests and analysis of typical portable electronic devices. International Journal of Mechanical Sciences, 50, 905-917. [17.6]

Zhou Q, Thomas MP, Stibich AM 1998. An analytical study of system variables for meeting FMVSS 201 head impact requirements. Crashworthiness, Occupant Protection and Biomechanics in Transportation Systems, Proceedings of the ASME (American Society of Mechanical Engineers) Congress, pp. 131-139. [4.5.1]

Zhu C, Wang D, Xia X, Tao Z, He M, Cao C 2018. The effects of gravel cushion particle size and thickness on the coefficient of restitution in rockfall impacts. Natural Hazards and Earth System Sciences, 18, 1811-1823. [17.11.4]

Zhu L, Liu Q, Jones N, Chen M 2018. Experimental study on the deformation of fully clamped pipes under lateral impact. International Journal of Impact Engineering, 111, 94-105. [17.12]

Zouggar K, Boukhoulda FB, Haddag B, Nouari M 2016. Numerical and experimental investigations of S-glass/polyester composite laminate plate under low energy impact. Composites, Part B, 89, 169-186. [17.15]

# Index

Topics are referred to by section number, or by chapter (Ch), Appendix (App), or Table (T).